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HIGHWAY SLAB-BRIDGES WITH CURBS: LABORATORY TESTS AND PROPOSED DESIGN METHOD

A REPORT OF AN INVESTIGATION
CONDUCTED BY
THE ENGINEERING EXPERIMENT STATION
UNIVERSITY OF ILLINOIS
IN COOPERATION WITH
THE PUBLIC ROADS ADMINISTRATION
FEDERAL WORKS AGENCY
AND
THE DIVISION OF HIGHWAYS
STATE OF ILLINOIS
BY

VERNON P. JENSEN
RALPH W. KLUGE
AND
CLARENCE B. WILLIAMS, JR.

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BY
VERNON P. JENSEN
Research Associate Professor of Theoretical and Applied Mechanics
RALPH W. KLUGE
Formerly Special Research Assistant Professor
of Theoretical and Applied Mechanics
AND
CLARENCE B. WILLIAMS, Jr.
Formerly Special Research Graduate Assistant
in Theoretical and Applied Mechanics

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HIGHWAY SLAB-BRIDGES WITH CURBS: LABORATORY TESTS AND PROPOSED DESIGN METHOD

I. Introduction

1. *Object and Scope of Investigation.*—The tests of slab-bridges with integral curbs described in this bulletin are intended to supplement the analytical studies previously reported in Bulletin 315.* Both studies form a part of an investigation of reinforced concrete slabs subjected to concentrated loads.

The theory on which the analytical study is based includes, besides the usual assumptions for reinforced concrete, certain approximations and assumptions concerning the interaction of the curb with the slab. The primary purpose of these tests, therefore, was to establish, if possible, the degree of approximation involved in the theory by comparing the actual behavior of the structure with that indicated by the theory or, specifically, by comparing measured strains at a number of points on the loaded structure with values computed from the mathematical analysis. Of importance also is a knowledge of the elastic and ultimate strength of the structure and the relation of these to design, as well as information concerning the type or manner of failure which may serve as a valuable guide to the proper design of details that lie outside the scope of the theory.

Tests were made on seven quarter-scale models and two half-scale models. The principal variables studied were the span length and the relative stiffness between curb and slab. In the prototypes the span varied from 10 to 30 feet. On one specimen handrails were included to determine their effect.

Following a study of the test results a simplified method of design was evolved. This method is broad enough in scope to include bridges with unlike curbs and spans up to 45 feet. When applied to the test structures the simplified method gives results which agree well with the tests.

The magnitudes of live loads have in all cases been determined in accordance with "Specifications for Highway Bridges" recommended by the American Association of State Highway Officials. Since the first test slab was designed and tested in 1939 the loadings were taken from the 1935 specifications which were in effect at that time. For all subsequent test structures, and for all analyses of the test structures, the loads were also taken in accordance with the 1935 specifications. Later,

*"Moments in Simple Span Bridge Slabs With Stiffened Edges," Univ. of Ill. Eng. Exp. Sta. Bul. 315, August 1939.

for illustrations of the simplified method of design, the magnitudes of live loads were taken in accordance with the 1941 specifications.

2. *Acknowledgment.*—The tests described in this bulletin form a part of an investigation being conducted by the Engineering Experiment Station in cooperation with the Public Roads Administration of the Federal Works Agency and the Illinois Division of Highways. The work was under the administrative direction of DEAN M. L. ENGER, Director of the Engineering Experiment Station, PROFESSOR F. B. SEELY, Head of the Department of Theoretical and Applied Mechanics, and PROFESSOR W. C. HUNTINGTON, Head of the Department of Civil Engineering.

The work of this investigation was performed under the general supervision of F. E. RICHART, Research Professor of Engineering Materials, University of Illinois, and was guided by an advisory committee consisting of the following representatives of the three cooperating agencies:

The Public Roads Administration of the Federal Works Agency:
E. F. KELLEY, Chief, Division of Tests
RAYMOND ARCHIBALD, Principal Structural Engineer;
The Illinois Division of Highways:
G. F. BURCH, Bridge Engineer
L. E. PHILBROOK, Assistant Bridge Engineer
(From 1936 until 1942, A. BENESCH, then Engineer of Grade Separations, served on the Advisory Committee);
The University of Illinois:
F. E. RICHART, Research Professor of Engineering Materials
N. M. NEWMARK, Research Assistant Professor of Civil Engineering
V. P. JENSEN, Research Associate Professor of Theoretical and Applied Mechanics.

Acting as consultants to the Advisory Committee are:
T. C. SHEDD, Professor of Structural Engineering
W. M. WILSON, Research Professor of Structural Engineering.

The tests reported herein embody the results of two separate projects. The first covered the tests of the two half-scale models, and was under the immediate direction of R. W. KLUGE. The second project covered the tests of the quarter-scale models, which were made and reported by C. B. WILLIAMS, JR.* Credit is due to L. E. GOODMAN,

*"Tests of Model Slabs With Curbs," C. B. Williams, Jr., thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in Civil Engineering in the Graduate School of the University of Illinois, 1941.

Special Research Graduate Assistant, who assisted in conducting many of the tests.

The simplified method of design described in Chapter IV was contributed by V. P. Jensen. Appreciation is expressed to Professors Newmark and Richart for valued criticisms and suggestions, and to other members of the Advisory Committee for a careful review of this study.

3. *Outline of Tests.*—The tests of the quarter-scale models were concerned principally with the variables, span length and relative stiffness between curb and slab. One group of specimens covered span lengths in terms of the prototype, of 10 ft., 20 ft., 25 ft., and 30 ft., each slab having curbs of ordinary dimensions, that is, of proportions commonly used for slab-bridges. In the other group of specimens the span was of constant length, 20 ft., and the stiffness of the curb varied over a fairly wide range, as represented by slabs with the following edge conditions:

(1) No curb
(2) Ordinary curb
(3) Heavy curb
(4) Ordinary curb with handrail.

The principal variable considered in the two half-scale models was the thickness of the slab, one slab being 2 in. thinner (4 in. thinner for the prototype) than the other. Both had spans of 12½ ft. corresponding to 25 ft. in the prototype. All of the test specimens represented two-lane bridges with a 24-ft. width of roadway.

The specimens were first tested in their initial uncracked condition, since at this stage the slabs more nearly conform to the homogeneous and elastic condition assumed in the theory. At normal working stresses, however, the concrete on the tension side of the section presumably cracks, and an entirely different set of conditions may ensue. Therefore, before proceeding to the second stage of the tests, the slabs were systematically cracked with a group of loads corresponding to the rear wheel concentrations of two trucks.

In the simply-supported slab-bridge the stresses governing the design of the structure are generally those occurring at the center of the slab, and at the mid-span of the curbs. However, the stresses in the slab near the curb were also investigated, as these become significant in a slab without curbs or with extremely small curbs. Most of the test observations, therefore, were confined to these three points in the

structure. The maximum live load moments at these points occur for wheel loads placed along the transverse centerline of the roadway. A convenient method of evaluating the maximum moments or stresses is by the use of influence lines. Influence ordinates for strain were therefore determined experimentally on all of the test structures for both the cracked and the uncracked slab and the values were compared with those computed from the theory. Strain corresponding to that produced by dead load was also measured as well as strain due to increasing multiples of the design live load placed in several positions on the slab. The structures were eventually tested to failure under a uniformly distributed load and a group of concentrated loads, corresponding to the live load, in a position to produce maximum moment at the center of the slab.

4. *Definition of Terms.*—The following terms are frequently used throughout this bulletin and are, therefore, defined and explained here:

(1) The *longitudinal axis* of the structure, also termed the *x axis,* is considered to be in the direction of the span and concurrent with the centerline of the roadway.

(2) The *transverse axis,* or *y axis,* is perpendicular to the x axis and is parallel to the supported edges of the slab.

(3) *Longitudinal unit strain,* ϵ_x, and *transverse unit strain,* ϵ_y, are strains in directions parallel to the x axis and y axis respectively.

(4) The *relative stiffness of the curb and slab* is denoted by the letter H, and is expressed by the equation

$$H = \frac{12\, I_{\text{curb}}}{ah^3}$$

wherein

I_{curb} = moment of inertia of gross section of curb
a = span of slab
h = overall thickness of slab.

(5) The value of b/a expresses the ratio of the width to the span of the slab. Following the practice recommended in Bulletin 315, b was taken, for the purposes of design, as the distance center to center of curbs. (A change in this practice is recommended for the simplified method of design described in Chapter IV.)

Special notation relative to the simplified method of design is defined in Chapter IV.

II. Description of Test Specimens and Apparatus

5. *Design of Test Specimens.*—Except for the reinforcement in the curbs of several specimens, the models were geometrically similar to full-sized slab-bridges which were designed for H-20 truck loadings as given in the American Association of State Highway Officials Specifications for Highway Bridges, 1935. With the exception of one specimen, handrails were not included, and in all specimens the wearing surface and roadway crown were omitted in the construction of the slabs. Design computations were based on the data given in Bulletin 315, although design stresses differed for several of the structures. The quarter-scale models were designed for working stresses of 1200 psi. and 20 000 psi. in the concrete and steel respectively, and one of the half-scale models, specimen 25BX, was designed for allowable stresses of 900 psi. and 18 000 psi. Because of the large margin of strength exhibited by the latter specimen, the other half-scale structure, specimen 25CX, was designed so as to produce a considerable reduction in the slab thickness. In this design a departure was made from the conventional procedure. Instead of designing for DL + LL and impact at given working stresses, as was done for the other test structures, this slab was designed to withstand DL + 2½ LL including impact, using a computed stress equivalent to the yield point of the reinforcing steel as the limiting value of stress. In other words, the structure was planned to have a theoretical safety factor of 2½ live loads before initial failure as determined by the yield strength of the reinforcing steel.

A summary of the design details for all of the test specimens and their prototypes is given in Tables 1 and 2, respectively. Diagrams showing the curb details of the various structures tested are presented in Fig. 1. The bar sizes and their spacing were to exact scale except in the curbs of the quarter-scale models, where this condition was impossible to fulfill in a practical manner. Consequently, in these details a scale ratio between the respective total steel areas was maintained rather than a ratio between actual bar sizes.

6. *Materials.*—With the exception of the ⅛-in. square bars the reinforcement was of intermediate grade, and the bars above ¼ inch in size were deformed. The ⅛-in. square bars were cold drawn SAE 1112 steel, and because of their extremely high yield point they were given a special heat treatment in order to reduce their yield strength to a value comparable to that of the intermediate grade bars. The physical properties resulting from this treatment are given in Table 3, which

TABLE 1
DESIGN SUMMARY FOR MODEL SLAB-BRIDGES

Specimen No.			10A	30A	20A	20B	20C	20D	25A	25BX	25CX
Scale of model			1/4	1/4	1/4	1/4	1/4	1/4	1/4	1/2	1/2
Span in feet			2 ft. 6 in.	7 ft. 6 in.	5 ft. 0 in.	5 ft. 0 in.	5 ft. 0 in.	5 ft. 0 in.	6 ft. 3 in.	12 ft. 6 in.	12 ft. 6 in.
Clear roadway width			6 ft. 0 in.	6 ft. 0 in.	6 ft. 0 in.	6 ft. 0 in.	6 ft. 0 in.	6 ft. 0 in.	6 ft. 0 in.	12 ft. 0 in.	12 ft. 0 in.
Type of curb			Ordinary	Ordinary	Ordinary	None	Heavy	Ordinary + Handrail	Ordinary	Medium heavy	Ordinary
Ratio of sides b/a			2.53	0.84	1.27	1.27	1.27	1.27	1.00	1.00	1.00
Relative stiffness H			0.98	0.16	0.32	0	0.86	0.32	0.23	0.58	0.32
Depth of slab (in.)			2 1/2	4 1/4	3 3/8	3 5/8	3 1/8	3 3/8	3 3/4	8 1/2	6 1/2
Slab Steel	Longitudinal	Bars	1/4-in. rd.–2 1/2 in. o.c.	1/4-in. rd.–1 15/16 in. o.c.	1/4-in. rd.–1 3/4 in. o.c.	1/4-in. rd.–1 1/2 in. o.c.	1/4-in. rd.–2 in. o.c.	1/4-in. rd.–1 3/4 in. o.c.	1/4-in. rd.–1 1/2 in. o.c.	1/2-in. rd.–3 in. o.c.	1/2-in. rd.–3 in. o.c.
		p	0.0093	0.0097	0.0094	0.0101	0.0090	0.0094	0.0097	0.0085	0.0114
	Transverse	Bars	1/8-in. sq.–1 3/4 in. o.c.	1/8-in. sq.–1 1/2 in. o.c.	1/8-in. sq.–1 1/16 in. o.c.	1/8-in. sq.–2 in. o.c.	1/4-in. rd.–6 in. o.c. 1/8-in. sq.–2 in. o.c.	1/8-in. sq.–1 1/16 in. o.c.	1/8-in. sq.–1 1/16 in. o.c.	1/2-in. rd.–8 in. o.c.	3/8-in. rd.–6 in. o.c.
		p_1	0.0042	0.0027	0.0036	0.0024	0.0049	0.0036	0.0032	0.0032	0.0032
	Top steel		Longitudinal—1/8-in. sq.–9 in. o.c. Transverse—1/8-in. sq.–4 1/2 in. o.c.							1/4-in. rd.–18 in. o.c. 1/4-in. rd.–8 in. o.c.	1/4-in. rd.–18 in. o.c. 1/4-in. rd.–9 in. o.c.
Curb	Top	Bars	None	4–3/8-in. rd.	3–1/4-in. rd.	No curb	4–1/4-in. rd.	3–1/4-in. rd.	3–3/8-in. rd.	6–1/2-in. sq.	3–1/2-in. sq.
		A_s'	0	0.44	0.15	....	0.20	0.15	0.33	1.50	0.75
	Bottom	Bars	6–1/8-in. sq.	5–1/4-in. rd. 1–3/8-in. rd.	5–1/4-in. rd.	No curb	1–3/8-in. rd. 3–1/4-in. rd.	5–1/4-in. rd.	6–1/4-in. rd.	7–1/2-in. sq.	5–1/2-in. sq.
		A_s	0.09	0.36	0.25	....	0.26	0.25	0.30	1.75	1.25

TABLE 2

DESIGN SUMMARY FOR PROTOTYPE SLAB-BRIDGES

Slab No.			10A	30A	20A	20B	20C	20D	25A	25BX	25CX
Span in feet			10	30	20	20	20	20	25	25	25
Type of curb			Ordinary	Ordinary	Ordinary	None	Heavy	Handrail	Ordinary	Med. heavy	Ordinary
Ratio of sides b/a			2.53	0.84	1.27	1.27	1.27	1.27	1.00	1.00	1.00
Relative stiffness H			0.98	0.16	0.32	0	0.86	0.32	0.23	0.47	0.28
Depth of slab (in.)			10	17	13½	14½	12½	13½	15	17	13
Slab Steel	Longitudinal	Bars	1-in. rd.–10 in. o.c.	1-in. rd.–5¼ in. o.c.	1-in. rd.–7 in. o.c.	1-in. rd.–6 in. o.c.	1-in. rd.–8 in. o.c.	1-in. rd.–7 in. o.c.	1-in. rd.–6 in. o.c.	1-in. rd.–6 in. o.c.	1-in. rd.–6 in. o.c.
		A_s/ft.	0.95	1.81	1.35	1.58	1.19	1.35	1.58	1.58	1.58
		p	0.0093	0.0097	0.0094	0.0101	0.0090	0.0094	0.0097	0.0085	0.0114
	Transverse	Bars	½-in. sq.–7 in. o.c.	½-in. sq.–6 in. o.c.	½-in. sq.–5¾ in. o.c.	½-in. sq.–8 in. o.c.	½-in. sq.–8 in. o.c. 1-in. rd.–24 in. o.c.	½-in. sq.–5¾ in. o.c.	½-in. sq.–5¾ in. o.c.	1-in. rd.–16 in. o.c.	¾-in. rd.–12 in. o.c.
		A_{s1}/ft.	0.43	0.50	0.52	0.38	0.64	0.52	0.52	0.59	0.44
		p_1	0.0042	0.0027	0.0036	0.0024	0.0049	0.0036	0.0032	0.0032	0.0032
	Top steel		Longitudinal—½-in. sq.–3 ft. 0 in. o.c. Transverse—½-in. sq.–18 in. o.c.							½-in. rd.–[a]3 ft. 0 in. o.c. ½-in. rd.–16 in. o.c.	½-in. rd.–3 ft. 0 in. o.c. ½-in. rd.–18 in. o.c.
Ratio A_{s1}/A_s			0.45	0.28	0.39	0.24	0.54	0.39	0.33	0.37	0.28
Curb Steel	Top	Bars	None	4–1⅛-in. sq. 2–1-in. sq.	3–1-in. rd.	No curb	4–1-in. rd.	3–1-in. rd.	4–1⅛-in. sq.	6–1-in. sq.	3–1-in. sq.
		A_s'	0	7.08	2.37		3.16	2.37	5.08	6.00	3.00
	Bottom	Bars	6–½-in. sq.	1–1¼-in. sq. 4–1-in. sq.	5–1-in. rd.	No curb	4–1-in. rd. 1–1-in. sq.	5–1-in. rd.	6–1-in. rd.	7–1-in. sq.	5–1-in. sq.
		A_s	1.50	5.56	3.95		4.16	3.95	4.74	7.00	5.00
Wheel load P, lb.			21 930	21 160	21 520	21 520	21 520	21 520	21 330	21 330	21 330
Unit load p, lb./ft.^{2}			125	212	169	181	156	169	188	212	163
Curb load q, lb./ft.			322	440	381	0	507	381	407	697	373
Moments per unit width of slab ft. lb./ft.	$(M_{slab})_{LL}$		10 830	21 880	16 400	19 370	14 900	16 400	19 030	17 130	18 510
	$(M_{slab})_{DL}$		1 460	19 730	6 960	8 930	5 370	6 960	11 940	11 640	9 920
	$(M_{slab})_{DL+LL}$		12 290	41 610	23 360	28 300	20 270	23 360	30 970	28 770	28 430
Moments in curb ft. lb.	$(M_{curb})_{LL}$		29 880	89 200	68 500	.....	104 300	68 500	81 200	119 700	91 700
	$(M_{curb})_{DL}$		7 820	100 400	40 500	.....	62 100	40 500	67 300	121 000	66 000
	$(M_{curb})_{DL+LL}$		37 700	189 600	109 000	.....	166 400	109 000	148 500	240 700	157 700

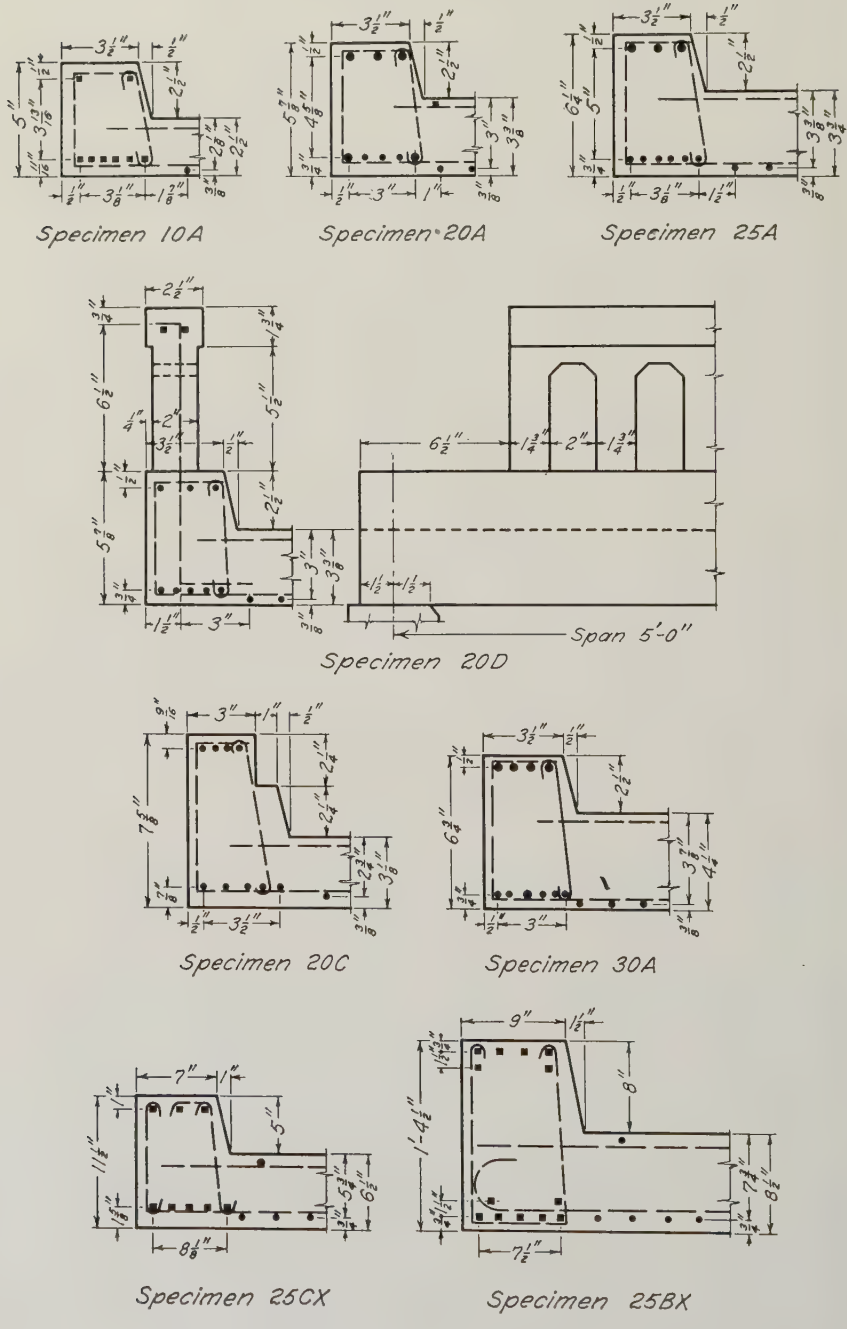


Fig. 1. Details of Curbs in Test Specimens

TABLE 3
PHYSICAL PROPERTIES OF STEEL
Values are the average of 6 tests

Bar	Yield Point psi.	Ultimate Strength psi.	Per cent Elongation in 2 in.	Per cent Elongation in 8 in.	Per cent Reduction in Area
⅛-in. square	41 000	59 000	27.5	....	59
¼-in. round	48 000	66 000	26.5	....	54
⅜-in. round	53 000	79 000	....	29.5	57
½-in. square	44 500	72 400	....	23.5	52
½-in. round	48 800	74 900	....	25.6	38

TABLE 4
PHYSICAL PROPERTIES OF CONCRETE
28-day tests—average of 6 cylinders

Specimen	Compressive Strength of 6 x 12-in. cyls. psi.	Modulus of Elasticity, E_c psi. x 10^6	Modular Ratio $n = E_s/E_c$
10A	3830	3.34	9.0
20A	4220	3.34	9.0
20B	3910	3.51	8.6
20C	3685	3.48	8.6
25A	4220	3.56	8.4
25BX	3420	3.70	8.1
25CX	3500	3.50	8.6
30A	3900	3.27	9.2

Note: Cylinders from 25BX and 25CX tested dry. All others coated with paraffin after 7-day period of moist curing.

also includes the properties of the other reinforcing steel used. After heat treatment the ⅛-in. bars were given an acid bath, with subsequent thorough washing, to remove the smooth protective scale formed on the surfaces of the bars. This process accelerated the formation of a light but firm and uniform coat of rust, which was necessary in order that the bars might develop sufficient resistance to bond failure.

The concrete used for all of the test specimens was designed to develop a strength of 3700 psi. at 28 days. Actual concrete strengths developed, with the corresponding moduli of elasticity, as determined from 6 x 12-in. control cylinders cured in the same manner as the slabs, are given in Table 4. The materials used were standard Portland cement, Wabash River Valley torpedo sand, and washed gravel. For the quarter-scale models the large aggregate consisted of pea gravel graded from No. 8 to ⅜ in. maximum size, and for the half-scale models the gravel was graded from ¼ in. to 1 in. maximum

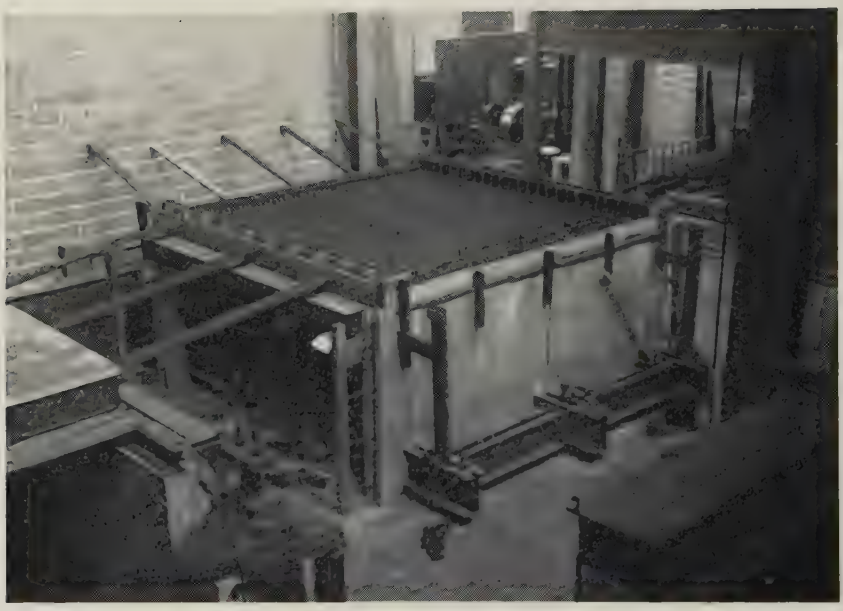


FIG. 2. VIEW OF REINFORCEMENT IN PLACE FOR SLAB 25CX

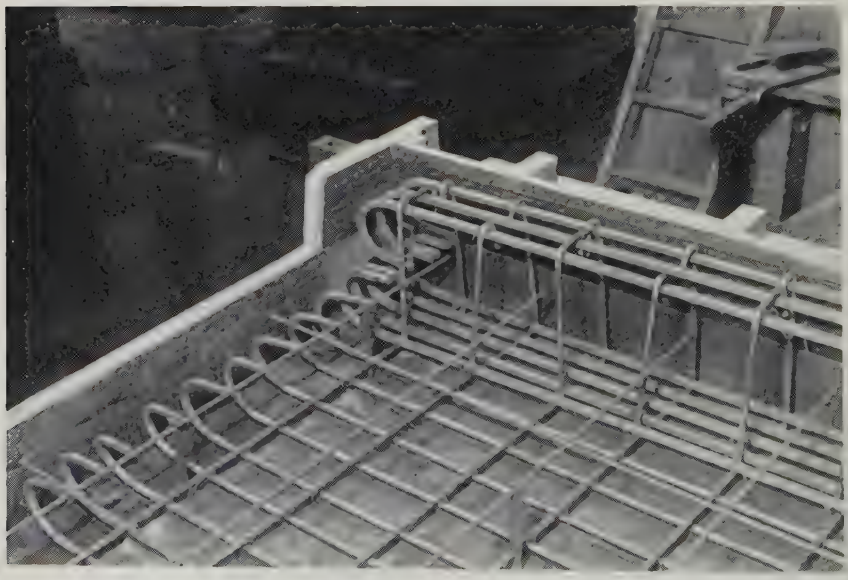


FIG. 3. VIEW SHOWING DETAIL OF REINFORCEMENT FOR SLAB 25CX

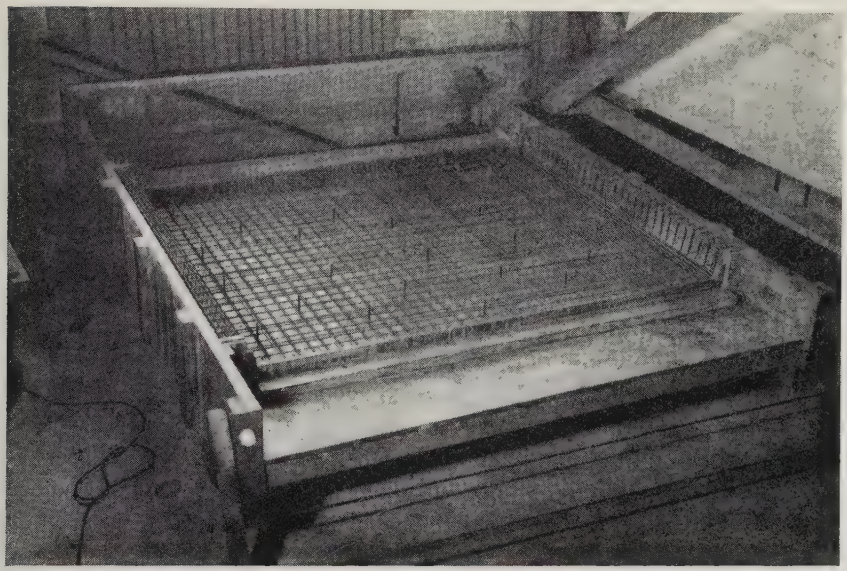


FIG. 4. VIEW OF REINFORCEMENT IN PLACE FOR SLAB 20C

size. The proportions of cement, sand, and gravel in the mix were 1:3.80:3.72 by weight, with a water-cement ratio of 1.2 by volume, for the quarter-scale structures, and 1:3.75:4.25 by weight, with a water-cement ratio of 0.95 to 1.0 by volume, for the half-scale structures. Both mixes produced a fairly stiff concrete with slumps varying from 1 to 3 in., and were placed by vibration.

7. *Construction of Test Specimens.*—The longitudinal and transverse reinforcing bars at the bottom of the slabs were securely wired together at frequent intervals and supported in the slab form by means of numerous vertical strips of steel, notched to receive the main steel at the bottom and the temperature steel near the top surface of the slab. Corks were fastened to the main and transverse bars at the center of the slab, and in the vicinity of the curb as well as on the top and bottom bars at the mid-span of the curb in order to expose the bars for strain measurements. Figures 2, 3, and 4 show views of the reinforcing steel in place for two of the slabs.

The concrete for the quarter-scale models was mixed for 3 minutes in a Lancaster mortar mixer of 2-cu. ft. capacity, whereas the concrete for the larger models was mixed for 3 minutes in a "Wonder" batch mixer of ⅕-cu. yd. capacity. The concrete was vibrated into place

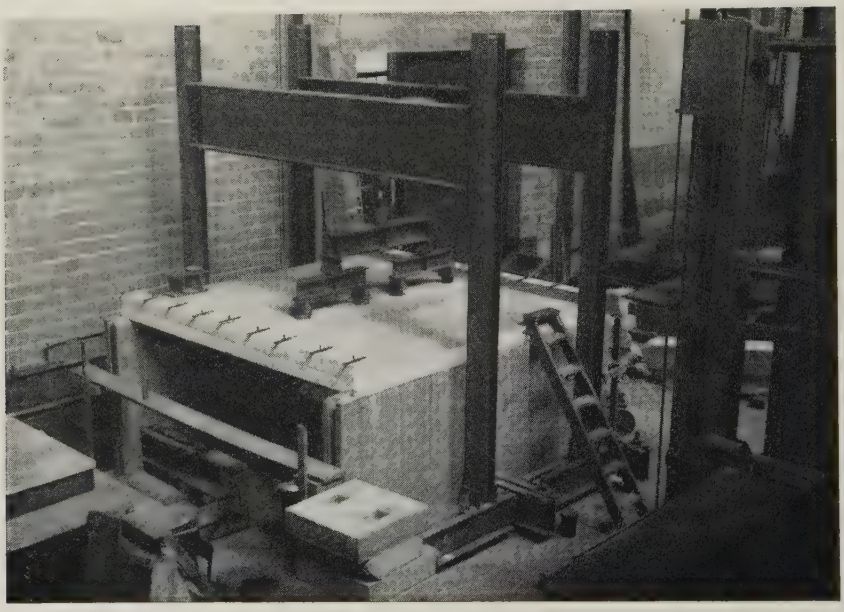


FIG. 5. VIEW OF LOADING APPARATUS FOR HALF-SCALE TESTS

with internal spud vibrators. Gravel having a maximum size of $\frac{3}{8}$ inch was substituted for the regular 1-in. size in the concrete placed around the lower layer of steel in the half-scale models because of the small clearance between the bars and the forms. The curbs were poured several hours after the slab forms were filled, and, in order to insure proper bond between the curb and slab, the vibrators were forced down through the curb into the concrete of the slab. After final set the exposed surfaces of the specimens were covered with wet burlap, and they were cured in this manner for 7 days; the half-scale models were then allowed to dry out in the laboratory air for an additional period of 21 days before testing began. Because of the relatively thin slabs of the quarter-scale structures, they were completely coated with a film of paraffin after the 7-day curing period, in order to prevent excessive shrinkage, with resultant curling at the corners. In this sealed condition they were then allowed to age an additional 21 days before being tested.

8. *Loading Apparatus.*—Load was applied to the test specimens by means of a screw-jack or a hydraulic-jack bearing against a structural steel frame, which, in the case of the half-scale model, was anchored to the laboratory floor, as shown in Fig. 5. For the quarter-scale speci-

FIG. 6. VIEW OF LOADING APPARATUS FOR QUARTER-SCALE TESTS

mens the loading frame was designed to transmit the jack reaction from the frame directly into the base of the abutment piers instead of through the medium of the laboratory floor. The piers could be shifted readily to accommodate the various span lengths, and the frame could be adjusted so as to permit the application of load at various points. A portion of this frame may be observed in Fig. 6.

In the design of slabs it is generally assumed that the wheel load of a truck is uniformly distributed over a circular area having a

diameter of 1.25 ft. for an H-20 loading. Loads were therefore transmitted to the test slabs through steel disks of a size corresponding to this diameter. For the quarter-scale and half-scale models their dimensions were 3¾ in. in diameter with a thickness of one inch, and 7½ in. in diameter with a thickness of 1¾ inches, respectively. In order to effect a more uniform distribution of the load over this area, the disks rested upon sponge-rubber mats. When group loads were applied to the slab a system of simply-supported beams resting on rockers distributed the jack pressure equally to four disks representing the rear wheels of two trucks as shown in Figs. 5 and 6.

Ring dynamometers of 10 000-lb., 50 000-lb., or 125 000-lb. capacity were used to measure the live load, except in the case of the test to failure of the half-scale models. For these tests the main load-distributing beam was calibrated and used as a dynamometer. The beam was equipped with a stiff frame pivoted on the lower flanges directly above its supports and contained a 1/10 000-in. Ames dial which measured the deflection at the center of the beam. Calibration data indicated a probable accuracy of ±500 lb. for loads above 20 000 lb. Loads measured with the rings were probably not more than one-half of one per cent in error in any case.

9. *Instruments.*—Strains in the quarter-scale models were measured in both concrete and steel by means of Huggenberger Tensometer gages which had a nominal multiplication ratio of about 1200 to 1, and operated over a 2-in. gage length. Their contact on the concrete surfaces was made through the medium of small brass plates, about $\frac{1}{32}$ inch in thickness, cemented to the concrete with a cellulose acetate cement.

Graphic gages* operating over a 6½-in. gage length were used to measure the concrete strains in the half-scale models. When properly used these gages can be depended upon to measure a strain of the order of 0.000005 in. per in. For the steel strains a 6-in. Berry gage capable of indicating strains of the order of 0.00003 in. per in. or better was used.

III. Description and Results of Tests

10. *Influence Ordinates for Strains in Uncracked Structures.*—Influence ordinates for strain in the concrete at the center of the slab and at the mid-span of the curbs were determined for a concentrated

*For a detailed description of the graphic gages see Bulletin 314, "Tests of Reinforced Concrete Slabs Subjected to Concentrated Loads," F. E. Richart and R. W. Kluge, University of Illinois Engineering Experiment Station, pp. 21-22.

load applied successively at intervals across the transverse centerline of the slab. For each position of the load, strains were observed in both top and bottom surfaces of the slab and curbs at the points indicated in Fig. 7. The load on the quarter-scale specimens was not in excess of 1000 lb. For the half-scale specimens, 25BX and 25CX, various loads were used. For specimen 25BX strain records were obtained at loads of 2000 and 4000 lb., and for specimen 25CX strain records were obtained at loads of 1500 and 3000 lb. These loads were believed to be as large as could safely be applied to the slabs without danger of cracking the concrete, and as far as could be observed no cracking occurred during this portion of the test.

The tests were repeated several times for each position of the load and the averages of the acceptable gage readings were reduced to unit strain per "wheel load." For this purpose a "wheel load" is defined as 1000 lb. on a quarter-scale model and 4000 lb. on a half-scale model. Either of these is equivalent to a load of 16 000 lb. on the full-scale structure and either one should produce a strain in the model equal to the strain produced in the full-scale structure by a load of 16 000 lb.

Influence lines for strain in the concrete at the mid-span of the slabs and curbs are shown in Figs. 8, 9, and 10. The smooth curves represent strains at the bottom surface of the slab and at the top and bottom of the curbs as computed from theoretical moments. The plotted points, connected by broken lines, represent measured values of strain. For convenience in making a comparison of measured and calculated strains at the center of the slab a slight adjustment was made in the measured strain at the top surface of the slab. This adjustment was made so that the measured strain would apply to an element at a distance from the neutral surface equal to the distance between this surface and the bottom of the slab. In this way a single theoretical curve serves for comparison with measured strains at both surfaces of the slab. The strains measured in the curbs were not adjusted.

From a study of Figs. 8, 9, and 10, the relative agreement between theoretical and observed strains may be seen to vary considerably for the slabs tested. The quarter-scale models show the greatest variability. Of these, slabs 10A, 20B, and 20C show reasonably good agreement whereas slabs 20A, 30A, 20D, and 25A show generally poor quantitative agreement. The half-scale models, 25BX and 25CX, show generally excellent agreement. There seems to be no consistent factor among the slabs tested, such as variation of span of slab or stiffness of curb,

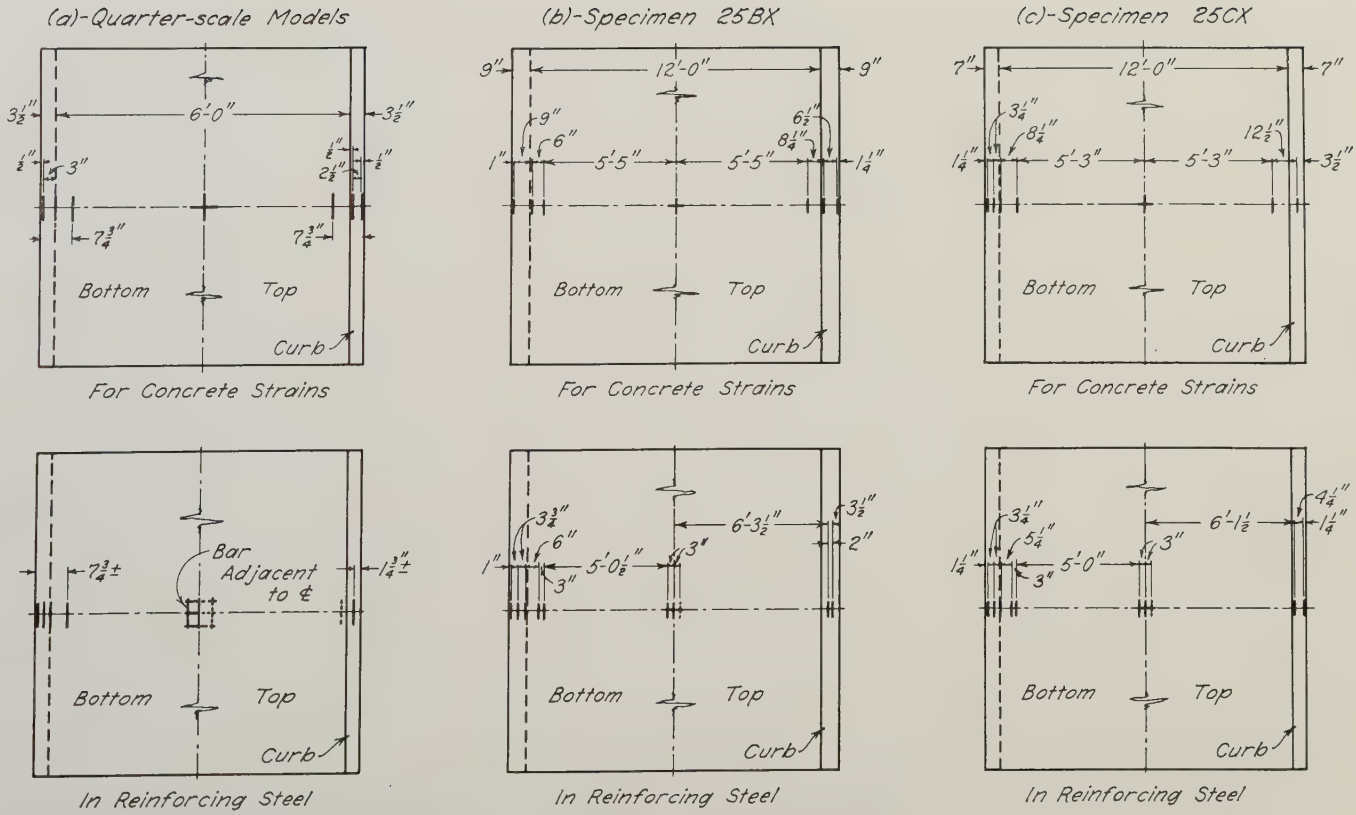


FIG. 7. DIAGRAMS SHOWING LOCATION OF GAGE LINES

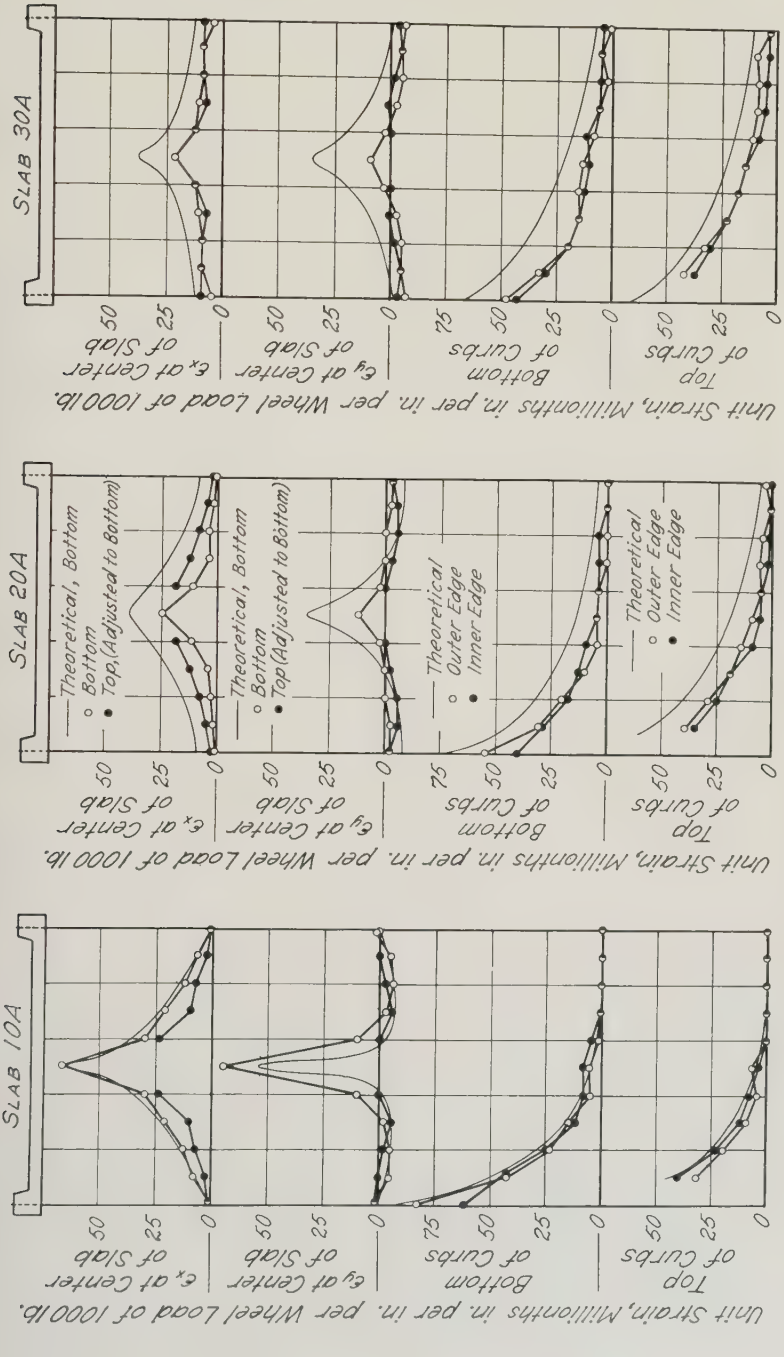


FIG. 8. INFLUENCE LINES FOR STRAIN IN UNCRACKED SLABS 10A, 20A, AND 30A

FIG. 9. INFLUENCE LINES FOR STRAIN IN UNCRACKED SLABS 20B, 20C, AND 20D

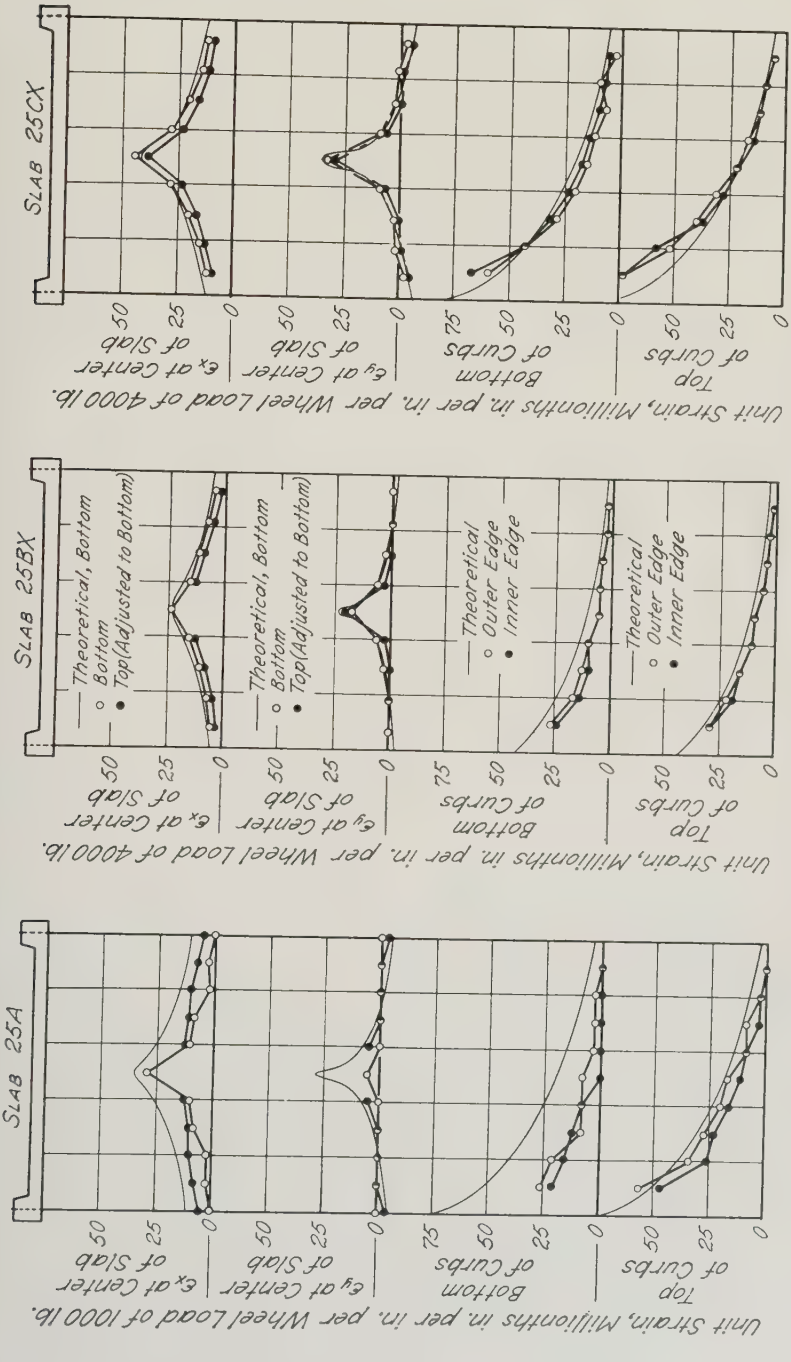

Slab 25A
Unit Strain, Millionths in. per in. per Wheel Load of 1000 lb.
Top of Curbs
Bottom of Curbs
ϵ_y at Center of Slab
ϵ_x at Center of Slab
Slab 25BX
Theoretical, Bottom
Bottom
Top(Adjusted to Bottom)
Theoretical
Outer Edge
Inner Edge
Unit Strain, Millionths in. per in. per Wheel Load of 4000 lb.
Slab 25CX
Unit Strain, Millionths in. per in. per Wheel Load of 4000 lb.

FIG. 10. INFLUENCE LINES FOR STRAIN IN UNCRACKED SLABS 25A, 25BX, AND 25CX

to account for the large differences noted. It appears possible that difficulties of instrumentation may be accountable in the quarter-scale models, particularly when it is noted that, for the instruments used, the least reading corresponds to an ordinate of about 4 millionths in. per in. per "wheel load" of 1000 lb.

Probably the most that can be said for these tests is that theoretical calculations for influence lines for longitudinal and transverse strain at the center of the slabs and for longitudinal strain at the center of the curbs are verified quantitatively by the tests of the half-scale models, and only qualitatively by the tests of the quarter-scale models.

An interesting result of all of the tests was the generally small differences between strains measured near the inside and near the outside edges of the curbs. The values on one side were not consistently larger or smaller than those on the other side. This is of particular interest because interaction between the curb and slab introduces, in addition to torsional effects, a state of unsymmetrical bending in the curbs, which presumably would result in appreciable differences between the strains on the inside and outside edges of the curb.

11. *Cracking of Test Specimens.*—Following the tests on the uncracked structure each slab was loaded with a group of four concentrated loads corresponding to the rear wheel concentrations of two trucks in adjacent lanes. These loads were applied at several positions along the transverse centerline, and then the loading procedure was repeated with the loads on lines from 6 to 24 in. on each side of the centerline and parallel to it.

The magnitudes of the loads applied to the various models are indicated in the following tabulation:

Specimen	Scale	Load per Rear Wheel on Each of 4 Wheels for 1 LL on Model,* lb.	Number of Live Loads on Model During Cracking Process	Percentage of Design Dead Load Acting During Cracking Process
10A	¼	1370	2.5	25
20A, B, C, D	¼	1345	2.5	25
25A	¼	1333	2.5	25
25BX	½	5300	2.25	50
25CX	½	5300	1.0	100
30A	¼	1323	2.5	25

*Including allowance for impact (1935 Specifications, A.A.S.H.O.).

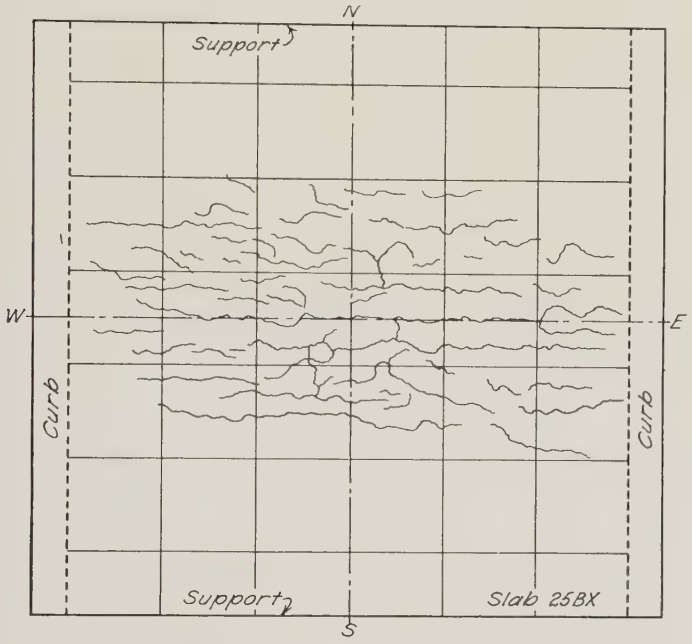


FIG. 11. SKETCH SHOWING PATTERN OF CRACKS IN BOTTOM OF SLAB 25BX AFTER INITIAL CRACKING

In all of the quarter-scale models 2.5 live loads were applied to the slab. In these models no additional dead load was applied at this stage of the test to make up for the deficiency in dead weight due to the scaling down of the slabs without a corresponding increase in the unit weight of the materials used. As a result, only 25 per cent of the design dead load was acting on the quarter-scale models during the process of cracking.

On the half-scale specimen 25BX no additional dead load was applied and 2.25 live loads (12 000 lb. per wheel) were applied to the slab. Cracking underneath the slab of specimen 25BX was quite general, most of the cracks running in a transverse direction across the slab, as shown in Fig. 11. It is characteristic of the slabs tested for the transverse cracks to coincide roughly with the bottom transverse reinforcing bars.

On the half-scale specimen 25CX the dead load was first brought to its full design value by applying a distributed load equal to the weight of the specimen. Then a single live load, consisting of four wheel loads, was applied to the slab in various positions to produce cracking. The only visible cracks resulting from this loading procedure were three fine transverse cracks, one along the centerline, and one on

FIG. 12. INFLUENCE LINES FOR STRAINS IN CONCRETE AND STEEL IN SLABS 10A, 20A, AND 30A AFTER CRACKING

each side parallel to it and about a foot away, all extending nearly to the curbs. However, before the subsequent load–strain tests were made on this slab it was more generally cracked due to the heavy single concentrated load used in the tests to determine the influence ordinates described in the following section.

On the quarter-scale specimens the paraffin coating obscured all but the relatively large cracks, but it is probable that the crack patterns were similar to that shown in Fig. 11.

During the cracking process no strain readings were made. However, it is characteristic of reinforced concrete that a residual tensile strain is left in the reinforcement as a result of the cracking of the concrete in tension under load and the subsequent removal of the load. The amount of such residual strain will depend on the severity of cracking. As a result of this phenomenon, strains measured subsequently to cracking may not be regarded as the total strains produced during the program of testing.

12. *Influence Ordinates for Strains in Cracked Structures.*—The procedure for determining the influence ordinates for the cracked structure was similar to that described in Section 10. The magnitude of the single concentrated load was 2000 lb. for the quarter-scale models, and varied for the half-scale models. For specimen 25BX influence ordinates were obtained with the slab in its initially-cracked condition and with a wheel load of 25 000 lb. After this the slab was subjected to 4.5 live loads at a number of positions and a new set of influence ordinates was obtained using a wheel load of 30 000 lb. For specimen 25CX a dual series of tests was also made, first with a wheel load of 12 000 lb., on the slab as initially cracked, and second, with a wheel load of 20 000 lb. applied after the slab had been subjected to 2.5 live loads.

Strains were measured on three longitudinal bars and one transverse bar at the center of the slab and on several bars at the bottom of each curb. Compressive strains were also observed in two bars at the top of the curbs of the half-scale specimens. Concrete strains were observed in both the longitudinal and transverse directions at the center of the slab and at the top of the curbs. A diagram indicating the positions of the gage lines is shown in Fig. 7. Gage readings were obtained from duplicate tests, and the average values were reduced to unit strain per "wheel load" as described in Section 10.

Experimentally-determined influence ordinates for strain are shown in Figs. 12, 13, and 14. For the quarter-scale models the ordinates

Slab 20B
Unit Strain, Millionths in. per in. per Wheel Load of 1000 lb.
Top Concrete Near Edge of Slab
Bottom Steel Near Edge of Slab
εy at Center of Slab
εx at Center of Slab
Above Adjacent Bar
Above Edge Bar
Edge Bar
Adjacent Bar
Slab 20C
Unit Strain, Millionths in. per in. per Wheel Load of 1000 lb.
Concrete Top of Curb
Steel Bottom of Curb
εy at Center of Slab
εx at Center of Slab
Outer Edge
Inner Edge
Outer Bar
Inner Bar
Bottom Steel
Top Concrete
Slab 20D
Unit Strain, Millionths in. per in. per Wheel Load of 1000 lb.
Concrete Top of Handrail
Steel Bottom of Curb
εy at Center of Slab
εx at Center of Slab
Outer Bar
Inner Bar

FIG. 13. INFLUENCE LINES FOR STRAINS IN CONCRETE AND STEEL IN SLABS 20B, 20C, AND 20D AFTER CRACKING

FIG. 14. INFLUENCE LINES FOR STRAINS IN CONCRETE AND STEEL IN SLABS 25A, 25BX, AND 25CX AFTER CRACKING

FIG. 15. VIEW OF SLAB 25CX SHOWING DEAD LOAD APPLIED TO SLAB

obtained at the center of the slab with the load on the two sides of the slab were averaged. For all curbs, the values plotted represent an average of the strains observed in both curbs. If the influence lines determined after initial cracking are compared with the theoretical curves obtained for the uncracked section (Figs. 8 to 10), it may be observed that they differ only slightly in most instances. However, calculated influence ordinates for strain based on a fully-cracked section are, in general, about three times as great as the experimentally determined values for the slabs as initially cracked. The marked increase in measured strains due to additional cracking is shown by the results obtained for specimens 25BX and 25CX. Furthermore, it must be remembered that the strains represented in Figs. 12, 13, and 14 are all due to reapplied loads, and do not include the residual strains resulting from previous applications of load.

As in the previous tests on the uncracked slabs, this series of tests may be said to give qualitative verification of the calculated influence lines for strains at the center of the slabs and at the mid-span of the curbs. Cracking was predominantly transverse to the span of the slabs.

The effect of interaction between the slab and curb is to lower the neutral axis in the curb and to raise it in the slab adjacent to the curb. For purposes of calculation the neutral axis of the curb may be assumed to be horizontal across the mid-span of the curb. The position of the neutral axis at the center of the slab is practically unaffected by the curbs.

13. *Tests Under Distributed Loads.*—Tests were made using distributed loads to simulate the effect of the dead loads on the prototype structures. As mentioned in Section 11, the actual dead load of a model produces in the model only a fraction of the dead load strain in its prototype when the material of the model is the same as that of the prototype. The fraction is ½ for a half-scale model and ¼ for a quarter-scale model. Consequently, tests were made on each of the half-scale structures in which dead weights were distributed on the surface of the slab and curbs so as to produce an intensity of load equal to the weight of the specimens. Figure 15 shows the dead load in place on specimen 25CX. The measured strains due to the superimposed load were doubled to give the strains to be expected from the full dead load on the prototype structures.

On each of the quarter-scale models, a distributed load equal to three times the actual weight of the model was applied to the slab and to the curbs. The measured strains due to these loads were multiplied by ⅓ to obtain the strains to be expected from the full dead load on the prototype structures.

The strains measured in this series of tests, after conversion to full dead load effect, are plotted as the initial points of the load–strain curves determined in subsequent tests (see Figs. 21 to 23).

The difference between the computed dead load strains in the steel and the experimentally-determined values at the center of the slab and at the bottom of the curbs was large in every case, although the ratios between the two were fairly consistent. In the longitudinal direction at the center of the slab the measured values were from 28 to 42 per cent of the theoretical values, with an average of 36 per cent. In the transverse direction at the center of the slab the percentages varied from 9 to 35 per cent, with an average of 18 per cent. In the bottom of the curbs the measured dead load strains varied from 20 to 37 per cent of the calculated values, the average being 28 per cent. At the top of the curbs the percentage varied from 39 to 110 per cent, with an average of 69 per cent. The great difference between the results found at the top and at the bottom of the curbs reflects the error in

the assumption that the location of the neutral axis in the curb is unaffected by interaction with the slab.

14. *Live Load Tests.*—After the tests described in Sections 10 to 13 had been completed, the specimens were tested with increasing multiples of the design live load along with full dead load, which in addition to the weight of the structure included the uniformly-distributed load described in Section 13. The live load represents the rear wheel concentrations, including an allowance for impact, of two standard 20-ton trucks in adjacent lanes, and was applied to the slab through four steel disks having the dimensions and spacing indicated in Fig. 16. The magnitude of the load on each disk for one live load, including an allowance for impact, is given in Section 11 for each of the models tested. Four separate tests were made, two with the concentrated loads placed to produce the maximum moment at the center of the slab, and one with the loads in position to produce maximum moment in each curb.

Strains were observed in both the longitudinal and transverse directions in the steel at the center of the slab, in the longitudinal direction in the steel and concrete in the slab adjacent to the curbs, and in the steel and concrete at the mid-span of the curbs. Load was measured on the quarter-scale specimens with a ring dynamometer of 50 000-lb. capacity, and on the half-scale specimens with a dynamometer of 125 000-lb. capacity. The live load was applied in increments of one-half live load until the measured strain in the reinforcing steel approached a value corresponding to a stress of about 35 000 lb. per sq. in., a stress fairly well below the yield strength of the steel.

Load–strain diagrams are shown in Fig. 17 for strain in the bottom longitudinal reinforcement at the center of the slabs. Calculated strains are also shown for comparison. The calculations were based upon moments determined according to the procedure described in Bulletin 315. It may be noted that moments computed according to this procedure are based upon a value of Poisson's ratio of 0.15. Computed stresses in the reinforcement were determined from the moments by the usual formula

$$f_s = \frac{M}{A_s jd}$$

and computed strains were obtained by dividing f_s by E_s.

A comparison of the diagrams in Fig. 17 for slabs 10A, 20A, and 30A shows that the agreement between theory and experiment becomes

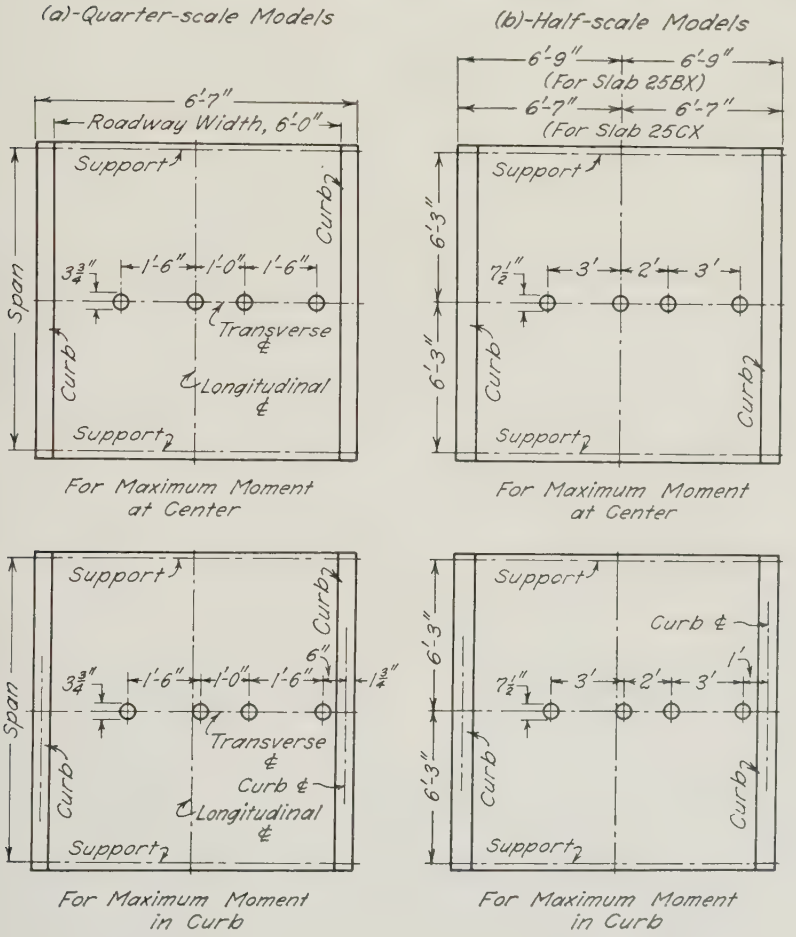
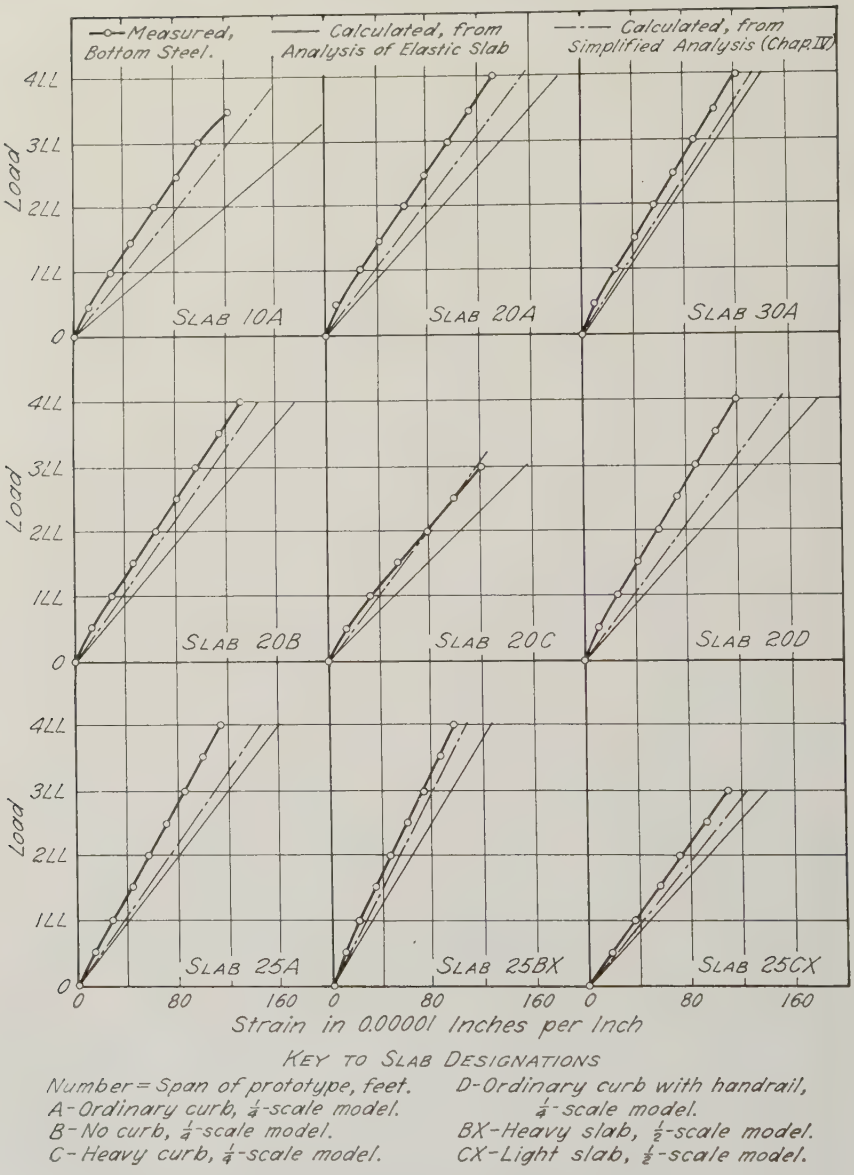


Fig. 16. Position of Loads for Maximum Moment at Center of Slabs and at Mid-Span of Curbs

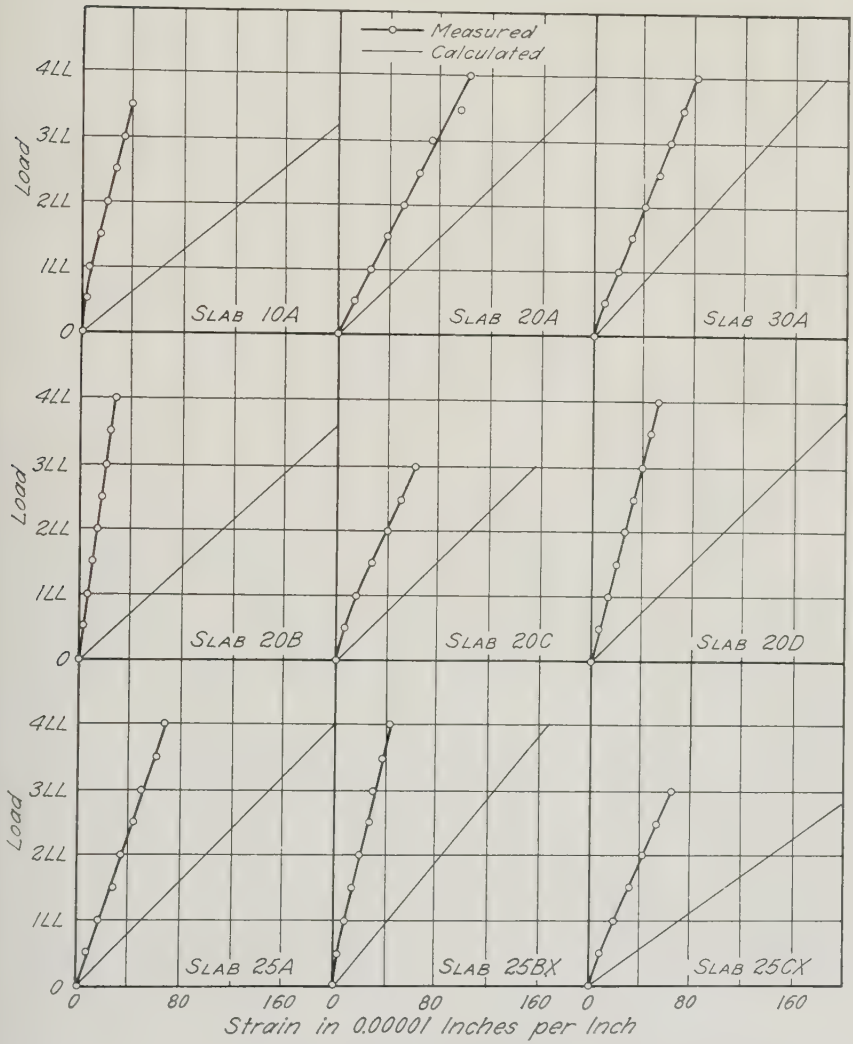
better as the span increases. Furthermore, all the slabs representing bridges of 20 or 25 ft. span show substantially similar results except possibly for slab 20D, in which the handrail may have caused some decrease in the measured strains. In all cases the measured strains are below the calculated values. The fact that slab 20B, without a curb, exhibits results similar to the other slabs may be interpreted to mean that there are factors present, such as tension in the concrete or redistribution of moments, which affect the agreement. These factors are in addition to the one which enters as a result of the choice of a method of computing the stiffness factor H.



KEY TO SLAB DESIGNATIONS

Number = Span of prototype, feet.
A - Ordinary curb, $\frac{1}{4}$-scale model.
B - No curb, $\frac{1}{4}$-scale model.
C - Heavy curb, $\frac{1}{4}$-scale model.
D - Ordinary curb with handrail, $\frac{1}{4}$-scale model.
BX - Heavy slab, $\frac{1}{2}$-scale model.
CX - Light slab, $\frac{1}{2}$-scale model.

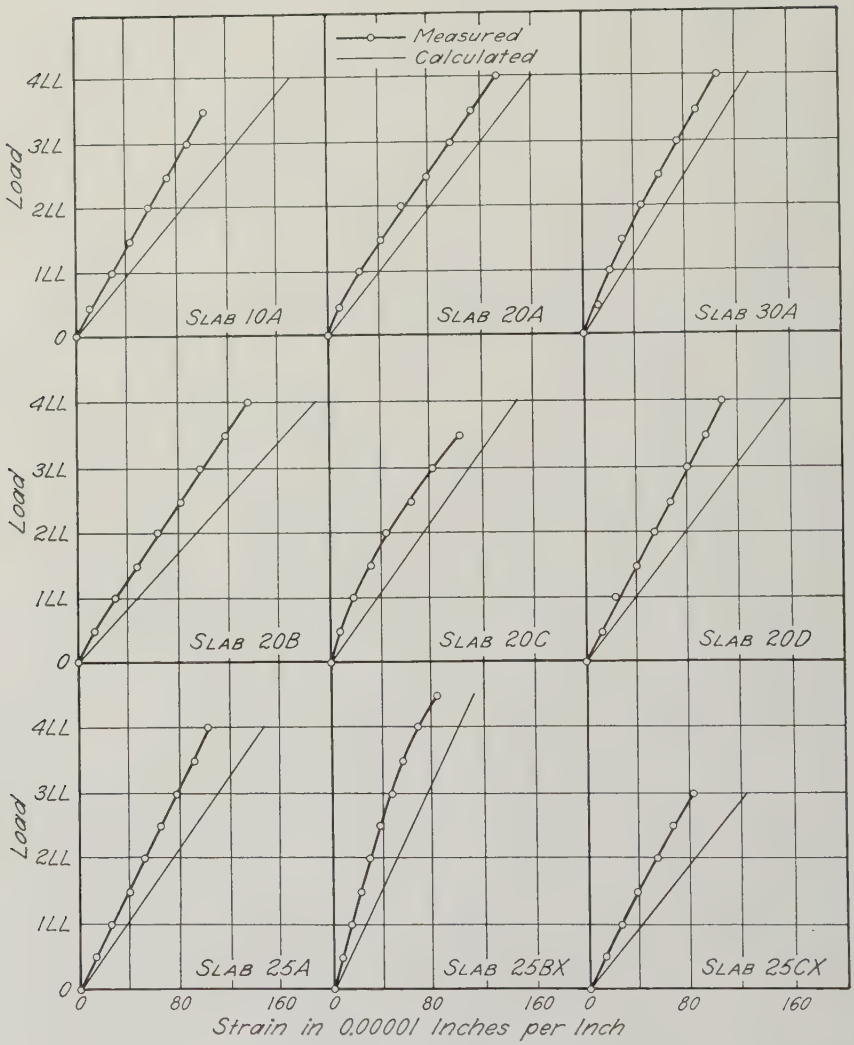
FIG. 17. LOAD–STRAIN DIAGRAMS FOR STRAIN ϵ_x IN LONGITUDINAL REINFORCEMENT AT CENTER OF SLABS



KEY TO SLAB DESIGNATIONS

Number = Span of prototype, feet.
A-Ordinary curb, $\frac{1}{4}$-scale model.
B-No curb, $\frac{1}{4}$-scale model.
C-Heavy curb, $\frac{1}{4}$-scale model.
D-Ordinary curb with handrail, $\frac{1}{4}$-scale model.
BX-Heavy slab, $\frac{1}{2}$-scale model.
CX-Light slab, $\frac{1}{2}$-scale model.

FIG. 18. LOAD–STRAIN DIAGRAMS FOR STRAIN ϵ_y IN TRANSVERSE REINFORCEMENT AT CENTER OF SLABS



KEY TO SLAB DESIGNATIONS

Number = Span of prototype, feet.
A-Ordinary curb, $\frac{1}{4}$-scale model.
B-No curb, $\frac{1}{4}$-scale model.
C-Heavy curb, $\frac{1}{4}$-scale model.
D-Ordinary curb with handrail, $\frac{1}{4}$-scale model.
BX-Heavy slab, $\frac{1}{2}$-scale model.
CX-Light slab, $\frac{1}{2}$-scale model.

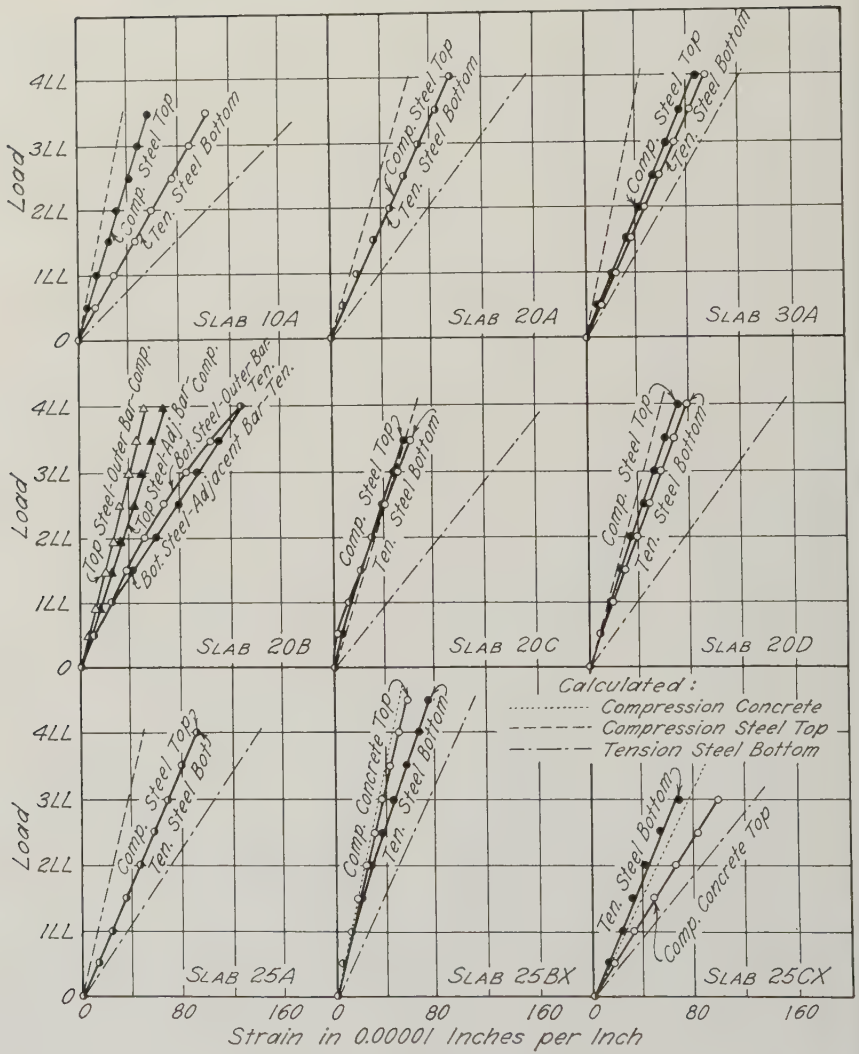
FIG. 19. LOAD–STRAIN DIAGRAMS FOR STRAIN ϵ_x IN LONGITUDINAL REINFORCEMENT IN SLAB NEAR CURB

There is no way of evaluating the exact extent of cracking of each of the slabs. However, it was believed that previous loadings had all been such as to keep the steel stresses below the yield point. It is well known from tests of beams that the calculated strain in the steel will ordinarily be below the measured strain until the yield point is reached. This means that the discrepancies shown on Fig. 17 should not be interpreted to mean that the moments developed in the tests are necessarily less than the computed live load moments at the center of the slabs. Furthermore, the residual strains due to initial cracking of the specimens are not included in the measured values shown on the diagrams.

Figure 18 shows the strains determined in the bottom transverse reinforcement at the center of the slab. These are all considerably below the calculated values based on a fully-cracked section. They are, however, greater than the values calculated on the basis of an uncracked section. The appearance of the bottom surfaces of the slabs showed that the cracks were generally normal to the main reinforcement, with only occasional cracks running normal to the transverse reinforcement. A comparison of Figs. 17 and 18 shows that the measured strains in the transverse steel, due to live load only, varied from 20 to 80 per cent of the measured strains in the longitudinal steel. It is probable that these percentages would not change materially with considerable change in the percentages of transverse steel in the slabs. In other words, it seems probable that the transverse strains would remain less than the longitudinal strains, even though a constant nominal percentage of transverse steel, say from 0.25 to 0.35 per cent, were used instead of percentages based on calculated transverse moment.

The strains shown in Fig. 19 were measured in the bottom longitudinal reinforcement of the slab near the curb with the loads in position to produce maximum moment in the curb. A comparison of these strains with those of Fig. 17 shows that the reinforcement in the slab near the curb is strained practically as much as the reinforcement at the center of the slab, except when the curb is exceptionally deep, as in specimen 20C.

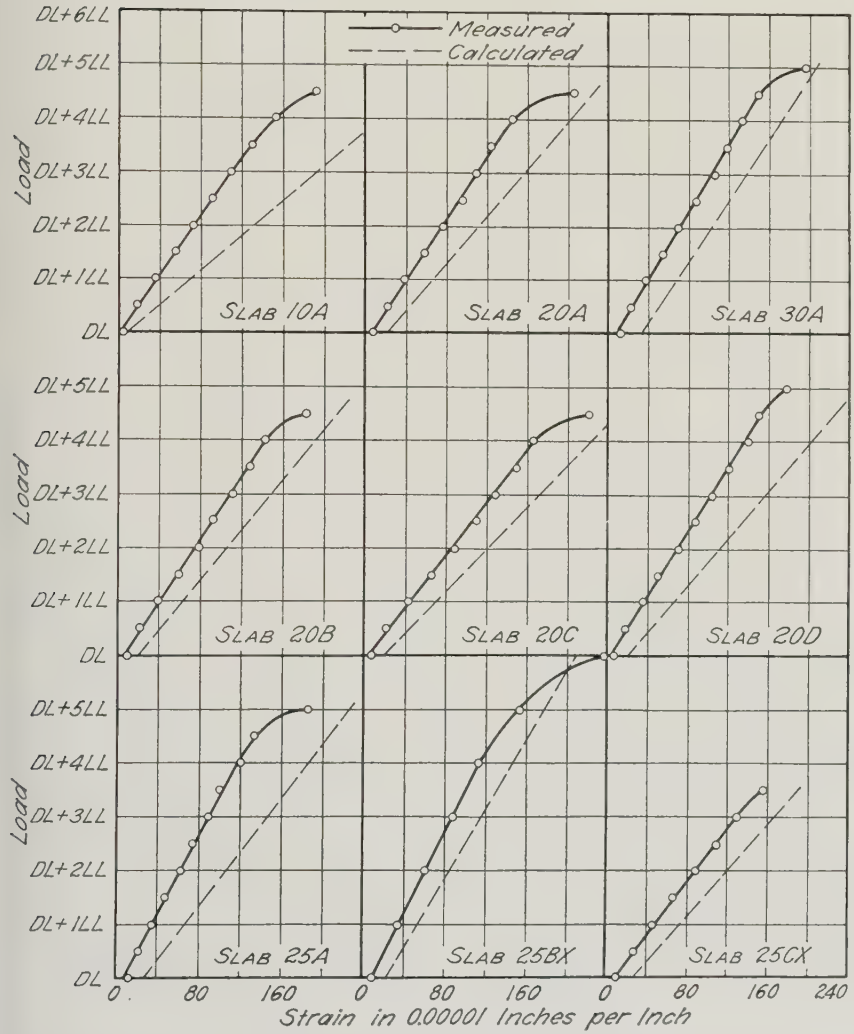
Diagrams representing measured strains at the mid-span of the curbs are shown in Fig. 20. It is evident from these diagrams that the position of the neutral axis in the curbs after cracking is affected by continuity with the slab. It would appear that the design of the curb should take account of this interaction with the slab.



KEY TO SLAB DESIGNATIONS

Number = Span of prototype, feet.
A-Ordinary curb, $\frac{1}{4}$-scale model.
B-No curb, $\frac{1}{4}$-scale model.
C-Heavy curb, $\frac{1}{4}$-scale model.
D-Ordinary curb with handrail, $\frac{1}{4}$-scale model.
BX-Heavy slab, $\frac{1}{2}$-scale model.
CX-Light slab, $\frac{1}{2}$-scale model.

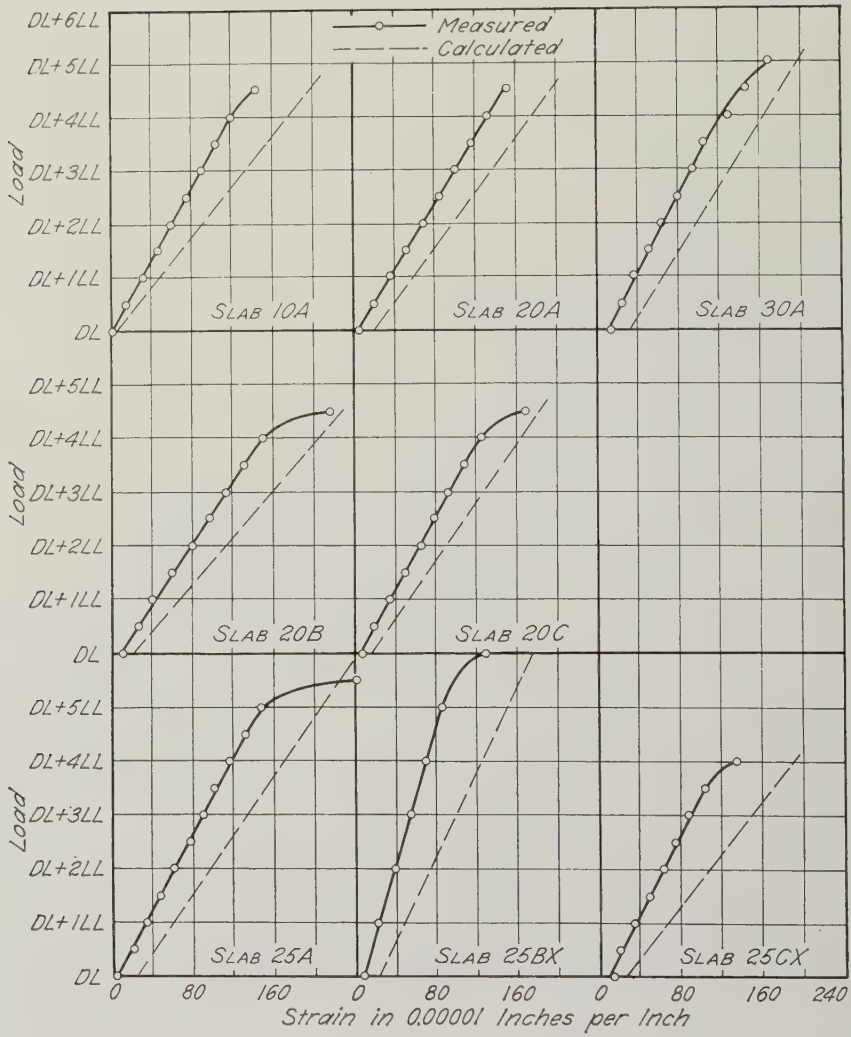
FIG. 20. LOAD–STRAIN DIAGRAMS FOR TENSION AND COMPRESSION AT MID-SPAN OF CURBS



KEY TO SLAB DESIGNATIONS

Number = Span of prototype, feet.
A-Ordinary curb, $\frac{1}{4}$-scale model.
B-No curb, $\frac{1}{4}$-scale model.
C-Heavy curb, $\frac{1}{4}$-scale model.
D-Ordinary curb with handrail, $\frac{1}{4}$-scale model.
BX-Heavy slab, $\frac{1}{2}$-scale model.
CX-Light slab, $\frac{1}{2}$-scale model.

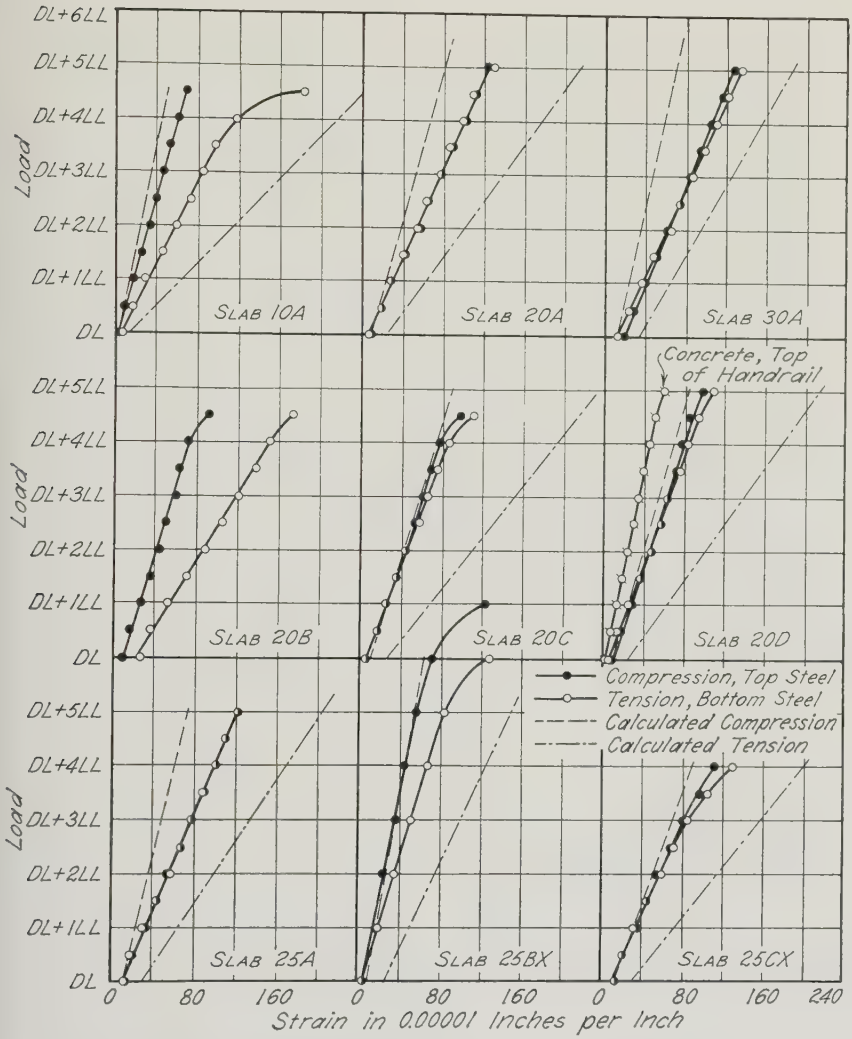
FIG. 21. CAPACITY TEST; LOAD–STRAIN DIAGRAMS FOR STRAIN ϵ_x IN LONGITUDINAL REINFORCEMENT AT CENTER OF SLAB



KEY TO SLAB DESIGNATIONS

Number = Span of prototype, feet.
A-Ordinary curb, $\frac{1}{4}$-scale model.
B-No curb, $\frac{1}{4}$-scale model.
C-Heavy curb, $\frac{1}{4}$-scale model.
D-Ordinary curb with handrail, $\frac{1}{4}$-scale model.
BX-Heavy slab, $\frac{1}{2}$-scale model.
CX-Light slab, $\frac{1}{2}$-scale model.

FIG. 22. CAPACITY TEST; LOAD–STRAIN DIAGRAMS FOR STRAIN ϵ_x IN LONGITUDINAL REINFORCEMENT IN SLAB NEAR CURB



KEY TO SLAB DESIGNATIONS

Number = Span of prototype, feet.
A-Ordinary curb, $\frac{1}{4}$-scale model.
B-No curb, $\frac{1}{4}$-scale model.
C-Heavy curb, $\frac{1}{4}$-scale model.
D-Ordinary curb with handrail, $\frac{1}{4}$-scale model.
BX-Heavy slab, $\frac{1}{2}$-scale model.
CX-Light slab, $\frac{1}{2}$-scale model.

FIG. 23. CAPACITY TEST; LOAD–STRAIN DIAGRAMS FOR TENSION AND COMPRESSION AT MID-SPAN OF CURBS

TABLE 5
SUMMARY OF DATA FOR TESTS TO FAILURE

Specimen*	Type of Curb	Scale of Model	Total Thickness of Slab, in inches		Slab Steel, per cent		Live Loads at Initial Failure			Live Loads at Final Failure	Ratio of Live Load at Final Failure to Live Load at Initial Failure
			Model	Prototype	Longitudinal	Lateral	Calculated	Test	Ratio $\frac{LL_{test}}{LL_{calc.}}$		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
10A	Ordinary	1/4	2 1/2	10	0.93	0.42	2.53	4.2	1.66	5.4	1.29
20A	Ordinary	1/4	3 3/8	13.5	0.94	0.36	3.09	4.3	1.39	5.5	1.28
25A	Ordinary	1/4	3 3/4	15	0.97	0.33	3.35	4.9	1.46	6.0	1.23
30A	Ordinary	1/4	4 1/4	17	0.97	0.27	3.64	4.7	1.29	6.0	1.28
25BX	Medium heavy	1/2	8 1/2	17	0.85	0.32	4.39	5.2	1.19	7.9	1.52
25A	Ordinary	1/4	3 3/4	15	0.97	0.32	3.35	4.9	1.46	6.0	1.23
25CX	Ordinary	1/2	6 1/2	13	1.14	0.32	2.89	3.5	1.21	4.7	1.34
20B	None	1/4	3 5/8	14.5	1.01	0.24	3.27	4.2	1.29	5.3	1.26
20A	Ordinary	1/4	3 3/8	13.5	0.94	0.36	3.09	4.3	1.39	5.5	1.28
20D	Ordinary with handrail	1/4	3 3/8	13.5	0.94	0.36	3.09	4.7	1.52	6.3	1.34
20C	Heavy	1/4	3 1/8	12.5	0.90	0.49	2.77	3.9	1.41	5.5	1.41

*Note: Span of prototype indicated by specimen designation; thus, 25A indicates 25-ft. span.

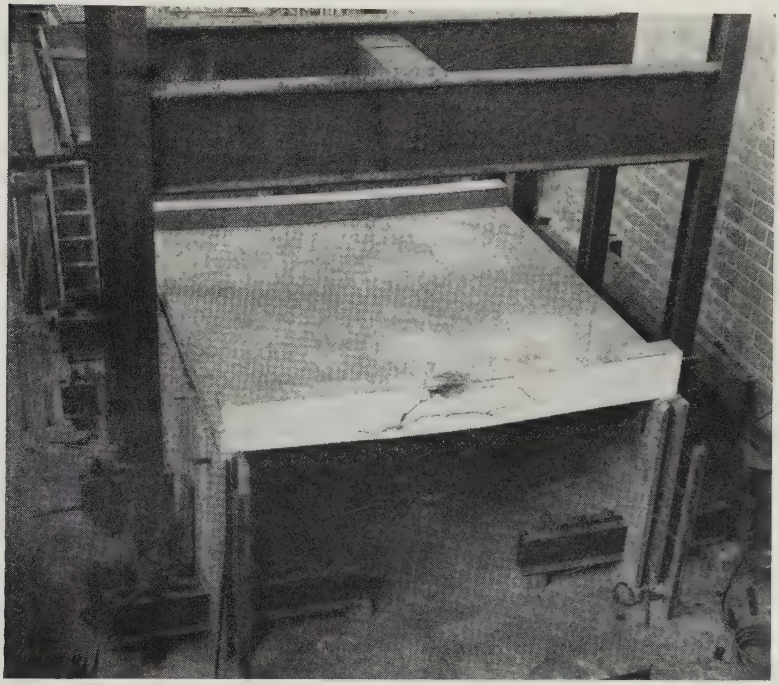


FIG. 24. VIEW OF SLAB 25BX AFTER TEST TO FAILURE

15. *Tests to Failure.*—Following the live load tests the slabs were loaded to failure with four concentrated loads representing the rear wheel concentrations of two trucks. The loads were spaced to produce maximum moment at the center of the slabs, with the exception of slab 25A, which was loaded for maximum moment in the curbs. In addition, each structure supported a distributed dead load as described in Section 13. Strains were measured at the center of the slab, in the slab adjacent to the curbs, and in the curbs at increments of one-half live load. Loads per wheel for one live load are indicated in Section 11.

Load–strain diagrams are shown in Figs. 21 to 23 for all of the slabs tested. In all of the specimens with curbs, first yielding was noted in the longitudinal steel at the center of the slab. In specimen 20B, which had no curb, the longitudinal steel apparently yielded first near the edge of the slab, with yielding at the center of the slab following at a slightly higher load.

Exact loads at first yielding are not known for several reasons. In the first place, initial stresses due to shrinkage and residual stresses

due to initial cracking are not known. In the second place, the measured strain depends upon the spacing of cracks relative to a gage line and upon the gage length. The greatest strain could be expected to be measured on a short gage length which is bisected by a crack. For comparative purposes the load at initial failure is defined as the load at which the measured strain in the steel was 0.0016 in. per in., which corresponds to a stress of 48 000 lb. per sq. in.

Loads at initial failure are indicated in Table 5, column 9. A comparison of specimens 10A, 20A, 25A, and 30A shows that all of these slabs carried over 4 live loads in addition to full dead load at initial failure. This is in contrast to the calculated number of live loads at yielding which varies from 2.5 to 3.6 because of the difference in ratios of design dead load to design live load for these slabs.

A comparison of live loads carried at initial failure of slabs 25BX, 25A, and 25CX shows that they all carried considerably more live load than the calculations indicated. The same may be said of slabs 20B, 20A, and 20C, in which the curbs varied. The slight excess of strength of slab 20D over slab 20A may possibly be attributed to the handrails.

Ordinarily a beam which fails by yielding of the steel will fail initially at a load which is near to the calculated load based on the ordinary straight-line theory of flexure of reinforced concrete. This is true even for a wide beam, provided that the moment is uniform across the width. For specimen 20B, which had no curb, the ratio of test to calculated live load at initial failure is 1.29. Erroneous distribution of moment between the curbs and slabs is not, then, the principal cause of the discrepancy between the theoretical and test load at initial failure.

If the resisting moment of slab 20B is calculated on the basis of an assumed uniform distribution of moment across the width of the slab, the result shows that 3.87 live loads plus full dead load are required to produce yielding of the steel. The test value of 4.2 live loads plus one dead load is not far above this figure. Since the prediction based on maximum moment is 3.27 live loads, it is apparent that the average moment is better than the maximum moment as a criterion of the carrying capacity of this slab at initial failure. If the same can be said for the slabs with curbs, the design procedure can be greatly simplified. This is discussed more fully in Chapter IV.

The ultimate load was determined for each slab as the load at which the deflection increased steadily with no increase in load when the jacks were operated continuously. The jacking arrangement in

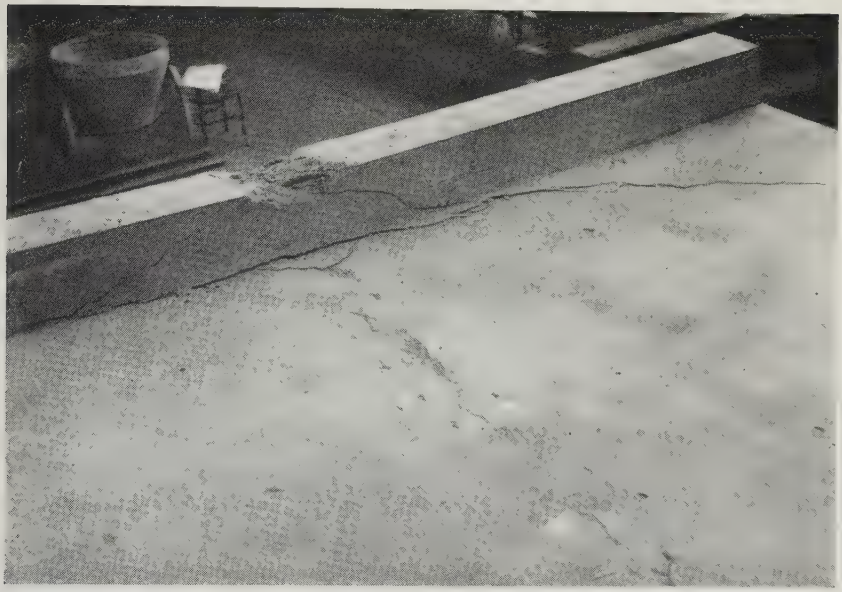


FIG. 25. CLOSE-UP VIEW OF TOP OF SLAB 25BX AFTER TEST TO FAILURE

the loading apparatus is described in Section 8. The ultimate loads for the half-scale models were, in addition to full dead load, 4.7 live loads for the thin slab of specimen 25CX, and 7.9 live loads for the heavy slab of specimen 25BX. All of the quarter-scale models, which were designed for 20 000 psi. in the steel and 1200 psi. in the concrete, carried between 5.3 and 6.3 live loads plus full dead load at final failure. For all slabs the ratio of live load carried at final failure to live load carried at initial failure varied from 1.23 to 1.52, as indicated in column 12 of Table 5.

The appearance of the slabs after testing may be judged in part by the photographs reproduced in Figs. 24 to 34, inclusive. Slab 25BX was the first to be constructed and tested. This slab was designed at more conservative stresses than any of the other slabs, and had a somewhat stiffer curb than any of the others except slab 20C. As already noted, primary failure occurred in slab 25BX by tension in the main steel at 5.2 live loads. Numerous transverse cracks opened up in the bottom of the structure. Ultimately spalling and crushing occurred at the top of the slab and curbs.

Stirrups in the curbs of specimen 25BX corresponded to U stirrups of ½-in. round bars at 2 ft. centers in the prototype. These stirrups

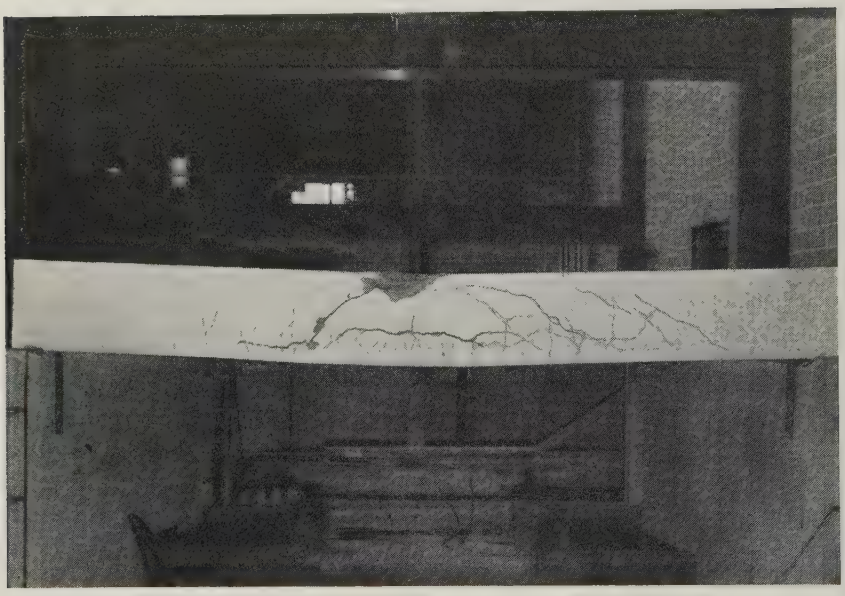


FIG. 26. SIDE VIEW OF CURB ON SLAB 25BX AFTER TEST TO FAILURE

were not designed on the basis of any calculated or assumed shear, but were merely put in as nominal web steel. That the stirrups were inadequate may be seen from the appearance of the specimen after failure, Figs. 24 to 26. The slab tended to pull away from the curbs, wide horizontal cracks appearing in the middle third of the span of the curbs. These cracks as well as diagonal tension cracks appeared after about 6 live loads had been applied. On the top of the slab negative bending caused tension cracks to open up in the neighborhood of the corners of the slab. These cracks, which were typical of all the slabs with curbs and which can be seen in Figs. 24 and 25, roughly outlined a circular region, dished downward, in the center of the slab.

Specimen 25CX was like 25BX in span and width of roadway. However, the total thickness of slab was reduced from 17 to 13 in., and the main steel was increased from 0.85 to 1.14 per cent. The curb was made smaller. Alternate transverse bars were bent upward into the outer and inner faces of the curb, and these were supplemented by additional vertical stirrups equal in area to the bent up bars.

For slab 25CX the primary failure by tension in the steel was similar to the primary failure of slab 25BX, although it occurred at

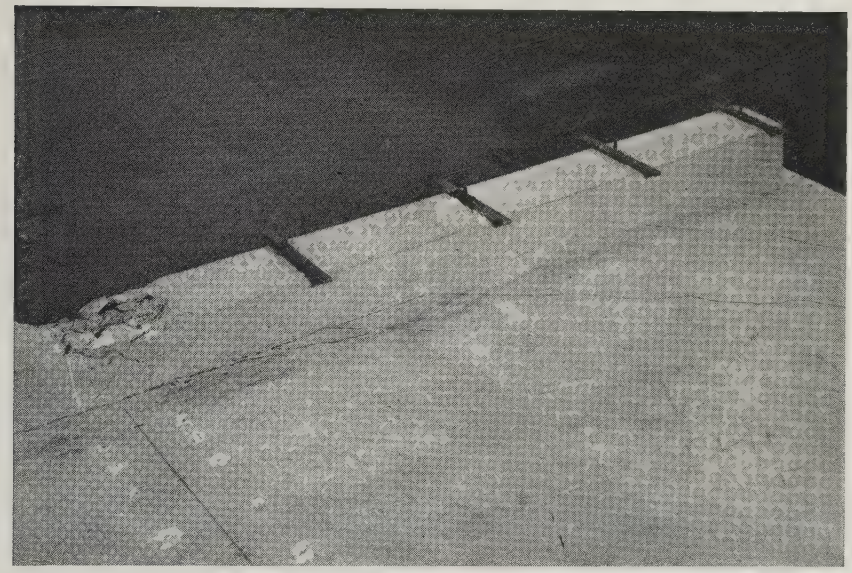


FIG. 27. CLOSE-UP VIEW OF TOP OF SLAB 25CX AFTER TEST TO FAILURE

an earlier load, as indicated in Table 5. The transverse steel picked up stress more rapidly after 3 live loads and probably yielded at a little more than 4 live loads, which was about the load causing yielding of the steel of the more highly stressed curb. The tendency for the slab to pull away from the curb was amply resisted by the web steel in the curb. A crack formed at the inside junction of the slab and curb at 4 live loads, but the crack remained small. No diagonal tension cracks formed in the curb. The cracks on top of the slab due to negative bending formed as the load was being increased from 4 to 4.5 live loads. Views of this specimen after failure are shown in Figs. 27 and 28. A secondary failure by crushing at the top of the curb, which occurred at 4.5 live loads, may be observed. The ultimate load carried by this slab was 4.7 live loads plus full dead load.

The general features of the tests of the quarter-scale models were similar to those of the half-scale models. All slabs failed initially by tension in the steel. Figure 29 shows the separation between curb and slab which ultimately took place in specimen 20C, which had the heaviest curb. This phenomenon was most noticeable in specimen 20C. The corner breaks due to negative bending may also be seen in the figure. This is a definite effect of the curbs, and was present in all slabs except 20B which had no curb. As the ultimate load was ap-

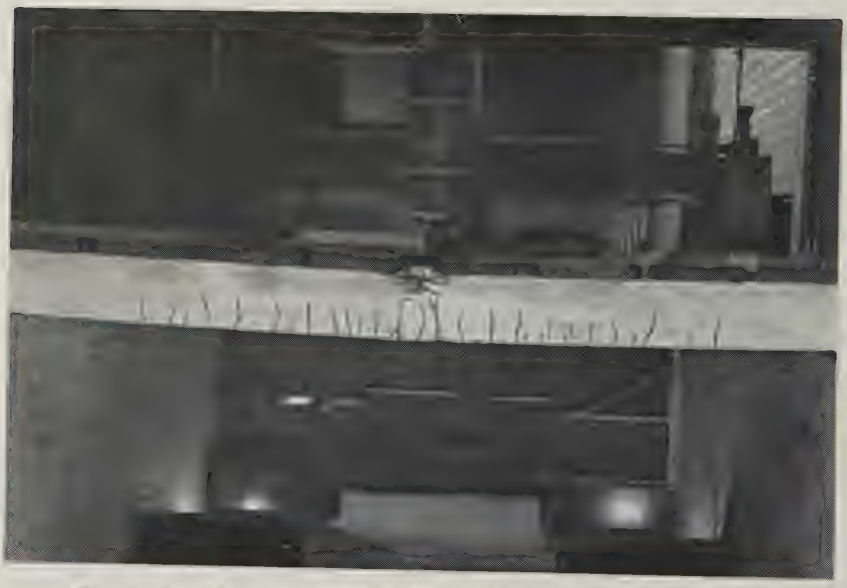


FIG. 28. SIDE VIEW OF CURB ON SLAB 25CX AFTER TEST TO FAILURE

FIG. 29. CLOSE-UP VIEW OF TOP OF SLAB 20C AFTER TEST TO FAILURE

proached it was noticed that the corners of slab 20B tended to lift off the abutments. This effectively relieved the negative bending near the corners.

Secondary crushing at the top of the curb is not evident in the photograph of specimen 20C (Fig. 29) although it did occur in other quarter-scale specimens.

The general appearance of the cracks on the bottom of several of the specimens may be seen in Figs. 30 to 33 inclusive. A wide crack opened up along the transverse centerline of each slab. Definite cracking in the longitudinal direction occurred near the center of all slabs except the one without curbs, but these cracks did not open widely. In Fig. 34 is shown specimen 20D, the slab with a handrail, after it had been tested. Separation of the spindles of the handrail from the curb and cracking of the spindles may be observed.

16. *Summary of Tests.*—Tests were made and are reported herein on nine models of reinforced concrete slab-bridges with curbs. Two of these were half-scale, and the remaining seven were quarter-scale models. Prototype spans varied from 10 to 30 ft., and curb details varied from no curb at all to a detail which was considered to produce an unusually stiff curb. Handrails were omitted except in one instance.

Designs were made in accordance with moments specified in Bulletin 315 for H-20 truck loads defined by the 1935 A.A.S.H.O. specifications. Calculations of moments are based on an analysis of a homogeneous elastic slab which is simply supported on two opposite edges and stiffened on the other edges by beams which deflect with the edges of the slab but are otherwise assumed to be unattached.

Tests on the uncracked half-scale models gave influence ordinates for longitudinal and transverse strain at the center of the slabs and for longitudinal strain at the center of the curbs which verified quantitatively the calculated values. Tests on the quarter-scale models gave, in general, only qualitative verification of the analysis. Figures 8, 9, and 10 show the principal results of these tests.

Influence ordinates for strain in the reinforcement obtained by tests on the cracked slabs were considerably smaller than the ordinates calculated for a fully-cracked section. Test values were shown to be markedly influenced by the extent of the cracking. The shape of the influence lines determined by test, and shown in Figs. 12, 13, and 14, again gave qualitative verification of the analysis.

Dead load strains, determined for distributed loads having an intensity equal to a fraction of the dead load and corrected to full

FIG. 30. VIEW OF BOTTOM OF SLAB 20C AFTER TEST TO FAILURE

FIG. 31. VIEW OF BOTTOM OF SLAB 20B (WITHOUT CURBS) AFTER TEST TO FAILURE

FIG. 32. VIEW OF BOTTOM OF SLAB 30A AFTER TEST TO FAILURE

FIG. 33. VIEW OF BOTTOM OF SLAB 10A AFTER TEST TO FAILURE

dead load, were less than the calculated values for all strains except those at the top of some of the curbs.

Tests were made under live load consisting of 4 loads representing the rear wheel concentrations of two trucks. Strains measured in the main reinforcement at the center of the slab showed increasingly better agreement with calculated values as the span increased and the maximum moment became more nearly equal to the average moment. The total discrepancy between theory and experiment cannot be accounted for by the action of the curbs, since the slab without curbs gave results similar to those obtained from the slabs with curbs. Factors such as extent of cracking and residual strain due to cracking affect the comparison of measured and calculated strain. The main

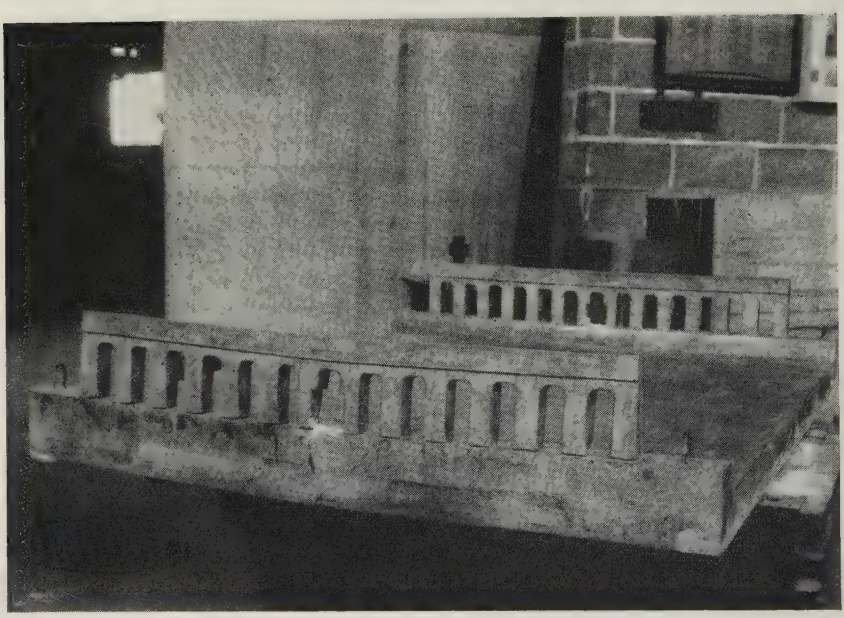


FIG. 34. VIEW OF SLAB 20D AFTER TEST TO FAILURE

reinforcement near the curb was found to be strained practically as much as the main reinforcement at the center of the slab. The transverse reinforcement was found to be strained much below the strain calculated on the basis of a fully-cracked section. Cracking normal to the transverse reinforcement was not pronounced. Measured strains in the curbs showed that the neutral axis in the curbs was materially lowered by interaction with the slab. Load–strain diagrams are shown in Figs. 17 to 20 inclusive.

The slabs were loaded to failure under full dead load and increasing multiples of live load placed to give maximum moment in the slab. Live loads were calculated with an allowance for impact as tabulated in Section 11. In all cases yielding of the main reinforcement appeared to be the initial cause of failure. First yielding occurred at the center of all slabs with curbs, and near the edge of the slab without curbs. Load–strain diagrams are shown in Figs. 21, 22, and 23. Loads at initial failure were considered to be those which produced a measured strain of 0.0016 in. per in. in the reinforcement. Such loads are compared in Table 5 with loads calculated to produce yielding of the steel. For the slab without curbs, the average bending

moment appears to be better than the maximum moment as an index of the capacity of the slab at initial failure. Failures of all structures were of the gradual yielding type, accompanied ultimately by secondary effects, such as crushing of the concrete of the slab and curbs, cracking along the inner junction between the curb and slab, and cracking on the top of the slab in the vicinity of the corners due to the action of the curbs in developing negative bending by holding down the corners of the slab. Ultimate live loads, given in Table 5, were in most instances about 25 or 30 per cent greater than the live loads at initial failure.

17. *Discussion of Results of Tests.*—Several of the principal features of the tests are singled out here for discussion. First, the capacity of the slab at initial failure checks the theoretical capacity more nearly as the span increases. The indication is that the average moment in a reinforced concrete slab is more indicative of its capacity than the maximum moment calculated by the theory of flexure for an elastic homogeneous slab. However, it so happens that the ratio of maximum moment to average moment is greatest for the slabs of small span, and that the ratio of live load to dead load moment is also greatest for the slabs of small span. The consequence is that the factor of safety, based on live load alone (defined as F where the total load carried at yielding of the reinforcement is DL + F LL), did not vary greatly with the span when the curb detail was kept constant and the designs were based on constant working stresses (see Table 5, column 9 for slabs 10A, 20A, 25A, and 30A).

In the following chapter a method will be described in which the slab may be designed on the basis of an average moment. However, it must be remembered that designs based on average moment for dead load plus one live load at working stresses will result in a variable factor of safety, whereas designs based on one dead load plus 2.5 or 3 live loads at yield point stresses will result in a constant factor of safety. Here the term factor of safety is understood to be based on live load only.

A second feature of the tests is that the transverse reinforcement was found to be greatly understressed. It would appear that the assumptions upon which the analysis is based are unsatisfactory in this instance. This is not surprising, since there is an essential contradiction in the method of analysis. This contradiction arises because the analysis is made for a homogeneous slab which, to satisfy geometrical

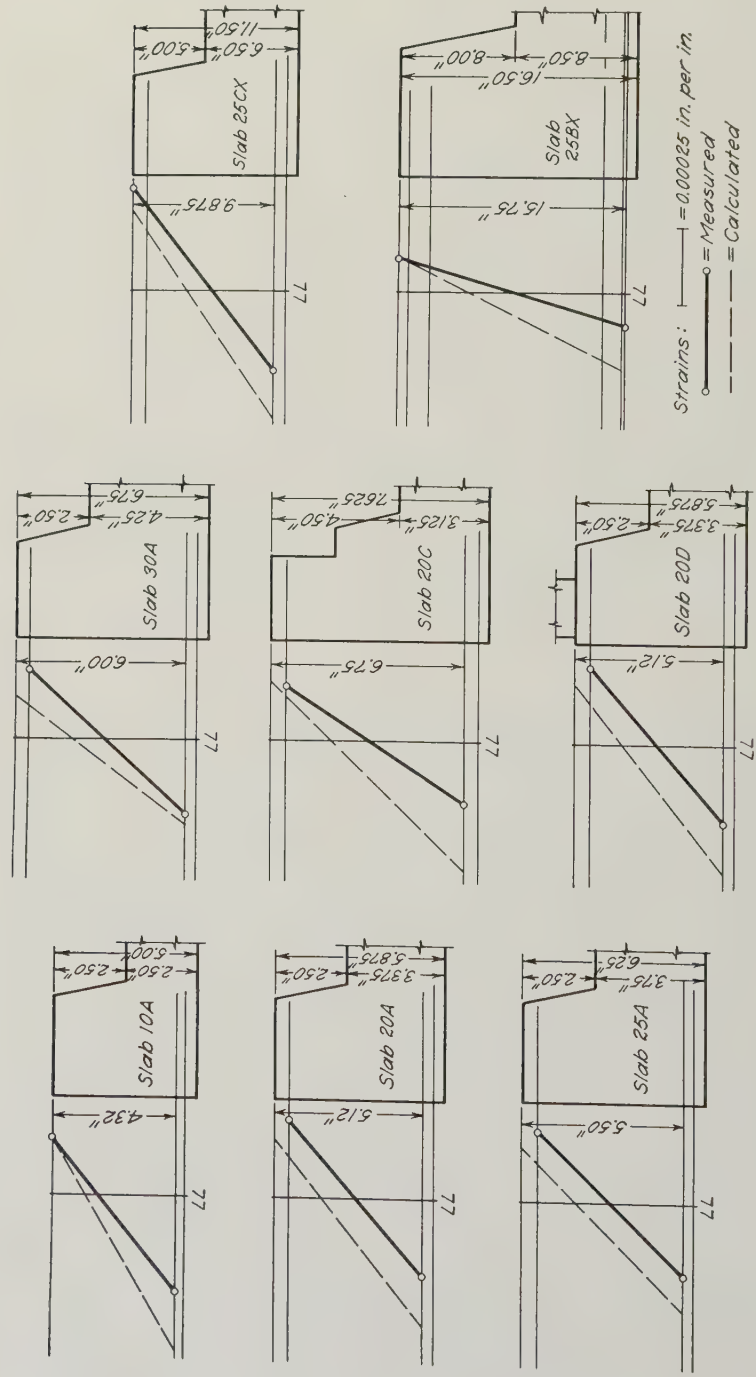


FIG. 35. MEASURED STRAINS IN CURB COMPARED WITH DESIGN CALCULATIONS

and statical requirements, is found to have different curvatures in the longitudinal and transverse directions. The bending moments accompanying these curvatures are also different in the two principal directions, and so the reinforcement is provided in different amounts in those directions with the implication that it will then be stressed alike. For this to happen the curvatures would become alike and the geometrical requirements would be violated. It appears that the curvature in the transverse direction remains less than the curvature in the longitudinal direction even for loads greater than those which produce yielding of the longitudinal steel. Transverse reinforcement is therefore forced to work less efficiently than the longitudinal reinforcement.

The question then arises as to whether the transverse reinforcement should be designed for a stress below the design stress for the longitudinal steel. Strictly speaking, it should if the concrete is not to be relied on for taking any tension. However, the tests demonstrate that the concrete does take tension, and cracking normal to the transverse steel is not as serious as cracking normal to the longitudinal steel. It is believed that a nominal percentage of bottom transverse reinforcement, say 0.35 per cent of bd, is adequate.

A third feature brought out by the tests is that the neutral axis in the curb is considerably lowered through interaction with the slab, and that the design of the curb is unsatisfactory when the effect of this interaction is neglected. Figure 35 graphically illustrates the effect of interaction on the strains at the top and bottom of the curbs and on the location of the neutral axes of the curbs at mid-span. Lines are drawn in the figure to show the calculated distributions of strain in the curbs due to live load when the curbs are treated as if severed from the slabs by a vertical cut at the inner faces of the curbs but are constrained to deflect identically with the edges of the slabs. Lines are also drawn to show the distribution of strain as determined from tests under live load only. Strains due to dead load have been omitted from the comparison because they are generally small, and their measurement is believed to be less reliable.

It will be observed that the general effect of interaction is to lower the neutral axis in the curb, to increase the compressive stresses in the top steel and in the concrete, and to decrease the stress in the tensile reinforcement. These effects result from the forces which are brought into action by the requirement that the longitudinal strains shall be alike in the curb and slab at the vertical section where they join. These forces may be visualized as distributed horizontal shears and twisting moments which act on the vertical face between the slab

and curb. Their action on the slab is to raise the neutral axis in the vicinity of the curb and to deepen the cracks in the slab which intersect the curb at right angles. These cracks may become so deep as to penetrate the entire depth of the slab. The effect of the horizontal shears and twisting moments on the curbs has already been noted.

The design of the curb is peculiar in the sense that the shape of the curb above the roadway surface is usually predetermined by safety requirements, type of handrail, and architectural considerations. Design consists only in the determination of the proper reinforcement. A simplified design procedure which accounts approximately for the interaction between the curb and slab is described in the next chapter.

Web reinforcement must be provided in the curb, particularly at the inner face where a stress concentration exists at the junction with the slab. Stresses in the web steel result in part from the resistance offered by the curb to the tendency of the slab to rotate at the edge. They result in part from the downward pull of the slab on the upper part of the curb due to live load. Such stresses, magnified at the reentrant corner, cannot be calculated by any of the usual formulas relating to the design of web reinforcement.

The quantity of web reinforcement in the curbs of all of the quarter-scale models was approximately 0.5 per cent of the horizontal cross-sectional area of the curbs immediately above the slabs. In these models the greatest distress resulting from horizontal cracking at the junction of the curb and slab occurred in specimen 20C, which had the stiffest curb. However, in this slab, as in all other slabs with curbs, cracking of this nature did not become serious until after the primary failure had occurred by yielding of the longitudinal reinforcement of the slab. In the half-scale models, specimen 25BX had practically no web steel, the actual quantity being 0.12 per cent, whereas specimen 25CX had about the same percentage as the quarter-scale models. The difference in the appearance of the curbs of these two half-scale specimens at ultimate failure has been noted previously (see Figs. 24 to 28 inclusive).

Probably no conclusion can be drawn from the tests as to the required percentage of web steel, except to say that none of the slabs failed primarily as a result of high web stresses, but that some web failures followed closely after initial tensile failures. One-half of one per cent of web reinforcement provided adequate resistance to diagonal tension failures in the slabs tested.

IV. Simplified Method of Design

18. *Moments for Design of Slab and Curbs.*—A procedure will be described herein for computing moments for the design of the slab-bridge with curbs. In this procedure the total bending moment to be resisted by the slab and curbs will be taken as the total moment obtained by statics plus a fractional increase in the live load moment in recognition of the fact that a sidewise shift of the loads will change the distribution of moment between the slab and curbs. The fraction of the total moment to be resisted by the curbs will be calculated from empirical equations based on the analysis given in Bulletin 315. The remaining part of the total moment will be assumed to be uniformly distributed over the width of the slab. As a result, the slab will be designed for an average moment rather than for the localized maximum moment calculated for a wheel concentration on an elastic homogeneous slab. The reinforcement in the curb will be determined for a composite section which includes a portion of the slab.

Loads and factors are as follows:

LL and DL = live load and dead load respectively; these may also appear as subscripts to denote live and dead load respectively

p = load per unit area between inside faces of curbs

q = load per unit length along curb; includes loads such as weight per unit length of structure outside of roadway width and load per unit length of sidewalk. Subscripts A and B refer to curb A and curb B respectively. For example, $(q_A)_{DL}$ refers to the dead load per unit length for curb A

P = magnitude of one rear wheel load including an allowance for impact

m = number of rear wheel loads on bridge ($m = 4$ for two-lane bridge).

The bridge is assumed to be simply supported, and the roadway is normal to the abutments. The dimensions and stiffness factors are as follows:

a = span of bridge center to center of bearing areas

b = width of roadway between inside faces of curbs

h = overall depth of the slab

v = axle width, center to center of truck tires; the analyses are based on $v = 6$ ft. for full-scale structures, $v = 3$ ft. for half-scale structures, etc.

s = distance between front and rear axles of truck and between adjacent fully loaded axles in truck-trailer (H-S) loadings; s = 14 ft. for full-scale structures

I = moment of inertia of the gross section of the curb outside of the roadway width

$G = \dfrac{1}{H} = \dfrac{ah^3}{12\,I}$ = dimensionless stiffness factor. (In this equation a, h, and I are to be expressed by the same dimensional units.)

$C_1, C_2, \ldots$ = dimensionless coefficients to be defined by empirical equations.

The span is limited to 30 ft. although with slight modification the analysis may be made applicable to longer spans. Such modification is discussed in Section 22. The width of roadway may be such as to accommodate two, three, or four lanes of traffic; the only requirement is that there be at least two lanes and that two rear wheel loads be applied per lane of traffic. The curbs need not be identical on the two sides of the bridge, although the degree of approximation becomes greater when the curbs differ. In the following discussion the curbs will be assumed to be alike, and modification will be given in Section 21 for bridges having unlike curbs.

The total bending moment to be resisted by the slab and curbs is known from statics. For maximum moment the wheel loads are on the transverse centerline of the bridge, and

$$M_{\text{statics}} = m\frac{Pa}{4} + \frac{pba^2}{8} + 2\frac{qa^2}{8}. \tag{1}$$

The contribution of each kind of load is easily recognized. For design the questions that need to be answered are these. How shall the moment contributed by each kind of load be apportioned to the slab and curbs? What kind of section shall be considered to resist the moment in the curb?

Empirical equations, based on the analyses described in Bulletin 315, are given as follows for the moment in the curb:

$$M_{\text{curb}} = C_1\frac{Pa}{4} + C_2\frac{pba^2}{8} + C_3\frac{qa^2}{8} \tag{2}$$

wherein

$$C_1 = \left(\frac{12}{2.5 + G}\right)\frac{4 - (v/a)}{4 + 28\,(v/a)} \tag{3}$$

$$C_2 = \frac{0.5\,(a/b)}{0.47\,G + \sqrt[3]{1.15 + (a/b)^3}} \tag{4}$$

$$C_3 = \frac{\sqrt[3]{1.15 + (a/b)^3}}{0.47\,G + \sqrt[3]{1.15 + (a/b)^3}}. \tag{5}$$

The stiffness factor G and the dimensions v, a, and b have been defined earlier. The ratios v/a and a/b are dimensionless and do not change their values as the scale changes in a model since all dimensions are scaled alike. The factor G likewise remains unchanged, and consequently the coefficients C_1, C_2, and C_3 have the same values for the prototype as for the model.

The moment to be resisted by the entire slab is the total moment given by Equation (1) minus the moment resisted by the two curbs after the wheel loads have been shifted somewhat toward the center of the bridge. The live load moment given by the first term in Equation (2) will be assumed to be reduced by 25 per cent when the loads are shifted. Thus, the moment to be resisted by the entire slab is

$$M = (m - 1.5\,C_1)\frac{Pa}{4} + (1 - 2C_2)\frac{pba^2}{8} + (2 - 2C_3)\frac{qa^2}{8}, \tag{6}$$

and the average moment per unit width of slab becomes

$$M_{\text{slab}} = (m - 1.5\,C_1)\frac{Pa}{4b} + (1 - 2C_2)\frac{pa^2}{8} + (1 - C_3)\frac{qa^2}{4b}. \tag{7}$$

This is the design moment for the slab when it is designed for dead load plus one live load at working stresses. If the design is to be based on dead load plus F live loads at yield point stresses, the quantity P in Equation (7) will be replaced by FP. In the latter case, F would be the expected factor of safety in terms of live loads at yielding.

For a particular design, the span a, the width of roadway b, and the number m and magnitude P of the wheel loads will be known.

The detail of the part of the curb above the roadway surface will also be specified. The first step in the design is to assume a depth of slab, h. Then one may calculate, in order, the moment of inertia of the gross section of the curb, I, the stiffness factor $G = ah^3/(12\ I)$, the coefficients C_1, C_2, and C_3, and finally the moment per unit width of slab according to Equation (7). The adequacy of the assumed thickness of slab may then be determined and revisions may be made if necessary. The reinforcement per foot of slab will be kept constant throughout the width of the slab.

To determine the quantity of reinforcement required in the curb an L-shaped section, including the curb and the adjoining slab of width $4h$, will be designed as a composite section. The moment in this composite section at the curb will be designated by a subscript L-curb on M. The bending moment to be resisted by the composite section will be the moment in the curb itself, given by Equation (2), plus $4h$ times the moment per unit width of slab given by Equation (7). Thus

$$M_{\text{L-curb}} = M_{\text{curb}} + 4h\ M_{\text{slab}}\ . \qquad (8)$$

Since the coefficients and loads have all been fixed by the final design of the slab, this moment may be calculated at once. It may be noted that $M_{\text{L-curb}}$ will be obtained in ft. lb. from Equation (8) when M_{curb} is expressed in ft. lb., h is expressed in ft., and M_{slab} is expressed in ft. lb. per ft.

It is not difficult to design the composite section at the curb, since the compressive stresses in the slab portion of the section, if any, may generally be neglected for a curb extending 9 or 10 in. above the roadway surface. Furthermore, the portion of the tensile steel in the width $4h$ is known from the design of the slab. It remains to determine the required compressive reinforcement, if any, and the remaining part of the tensile reinforcement.

In determining these steel areas it should be remembered that the actual position of the neutral axis in the curb is naturally low due to the action of the slab on the curb. Design according to the usual working stresses will not produce this low position of the neutral axis. Either the tensile stress in the steel must be decreased, or the compressive stress in the concrete must be increased, or both. It will be found that decreasing the working stress in the tensile reinforcement results in a lowering of the calculated position of the neutral axis, and produces increased efficiency of the compressive reinforce-

ment. The net result is that the total required area of steel is changed but little. However, excessive requirements of compressive reinforcement are avoided in this way, and the curb is permitted to act in a normal manner.

The 1940 Report of the Joint Committee on Standard Specifications for Concrete and Reinforced Concrete as well as the 1941 Building Regulations of the American Concrete Institute tacitly permit higher compressive stresses to be used in the concrete when compressive reinforcement is required in beams. This comes about through the provision that, with certain limitations as to maximum stress, the effectiveness of compression reinforcement in resisting bending may be taken at twice the value indicated from the calculations assuming a straight-line relation between stress and strain. Such a provision is an admission that excessive compressive stresses in the concrete are not serious when compression steel is present. Under the action of plastic flow an adjustment will take place; the concrete stresses will be relieved and the compressive stresses in the steel will be raised.

Transverse reinforcement in the slab need not be determined on the basis of calculated transverse bending moment but may simply be provided at a nominal percentage such as 0.35 per cent of bd.

19. *Analyses of Test Structures by Simplified Method.*—In order to evaluate the simplified method of calculating moments for the design of slab-bridges with curbs, the prototypes of the test structures were analyzed by this method, and a comparison was made of the measured and calculated stresses and of the load capacities. For the purpose of comparison the moments due to live load and dead load were calculated separately as follows:

$$(M_{\text{slab}})_{\text{LL}} = (4 - 1.5\,C_1)\frac{Pa}{4b} \tag{9}$$

$$(M_{\text{slab}})_{\text{DL}} = (1 - 2C_2)\frac{pa^2}{8} + (1 - C_3)\frac{qa^2}{4b}, \tag{10}$$

and

$$(M_{\text{L-curb}})_{\text{LL}} = C_1\frac{Pa}{4} + 4h\,(M_{\text{slab}})_{\text{LL}} \tag{11}$$

$$(M_{\text{L-curb}})_{\text{DL}} = C_2\frac{pba^2}{8} + C_3\frac{qa^2}{8} + 4h\,(M_{\text{slab}})_{\text{DL}}\,. \tag{12}$$

TABLE 6

DATA FOR CALCULATION OF MOMENTS IN PROTOTYPES OF TEST STRUCTURES USING SIMPLIFIED METHOD

$b = 24$ ft. for all slabs except 20B for which $b = 26.33$ ft.

Slab	Span a ft.	Overall Depth of Slab h in.	$G = 1/H = \frac{ah^3}{12\,I}$	Moment Coefficients			Live Load	Dead Load	
				C_1 Eq. (3)	C_2 Eq. (4)	C_3 Eq. (5)	P lb. per wheel	p lb. per sq. ft.	q lb. per ft.
10A	10	10	1.01	0.559	0.1349	0.692	21 930	125	322
20A	20	13.5	3.08	0.642	0.1574	0.453	21 520	169	381
25A	25	15	4.37	0.613	0.1545	0.391	21 330	188	407
30A	30	17	6.06	0.555	0.1451	0.339	21 160	212	440
20B	20	14.5	∞	0	0	0	21 520	181	0
20C	20	12.5	1.15	0.981	0.2394	0.689	21 520	156	507
20D	20	13.5	3.08	0.642	0.1574	0.453	21 520	169	381
25BX	25	17	2.11	0.913	0.2257	0.570	21 330	212	697
25CX	25	13	3.61	0.689	0.1729	0.437	21 330	163	373

Table 7
Moments in Prototypes of Test Structures Calculated by Simplified Method

Specimen No.	Moments in Slab						Moments in Composite Curb			Calculated Number of LL on Slab at Initial* Failure	Number of LL on Test Structures at Initial Failure
	$(M_{slab})_{LL}$		$(M_{slab})_{DL}$		$(M_{slab})_{LL+DL}$		$(M_{L\text{-}curb})_{LL}$ ft. lb.	$(M_{L\text{-}curb})_{DL}$ ft. lb.	$(M_{L\text{-}curb})_{LL+DL}$ ft. lb.		
	ft. lb./ft.	Ratio†	ft. lb./ft.	Ratio†	ft. lb./ft.	Ratio†					
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
10A	7 220	0.67	1 240	0.85	8 460	0.69	54 700	12 000	66 700	3.81	4.2
20A	13 610	0.83	6 660	0.96	20 270	0.87	130 300	70 500	200 800	3.74	4.3
25A	17 110	0.90	11 760	0.98	28 870	0.93	167 300	125 700	293 000	3.74	4.9
30A	20 950	0.96	19 660	1.00	40 610	0.98	206 800	211 200	418 000	3.81	4.7
20B	16 330	0.84	9 050	1.01	25 380	0.90	0	0	0	3.87	4.2
20C	11 340	0.76	4 720	0.88	16 060	0.79	152 800	82 000	234 800	3.69	3.9
20D	13 610	0.83	6 660	0.96	20 270	0.87	130 300	70 500	200 800	3.74	4.7
25BX	14 610	0.85	11 040	0.95	25 650	0.89	204 500	183 300	387 800	5.19	5.2
25CX	16 480	0.89	9 700	0.98	26 180	0.92	163 300	107 600	270 900	3.26	3.5

*Calculated for $Y.P.$ = 48 000 psi. in slab reinforcement and n = 9. Live loads are in addition to full dead load.
†Ratio of the bending moment calculated by the simplified method to the bending moment calculated by the method described in Bulletin 315 and given in Table 2.

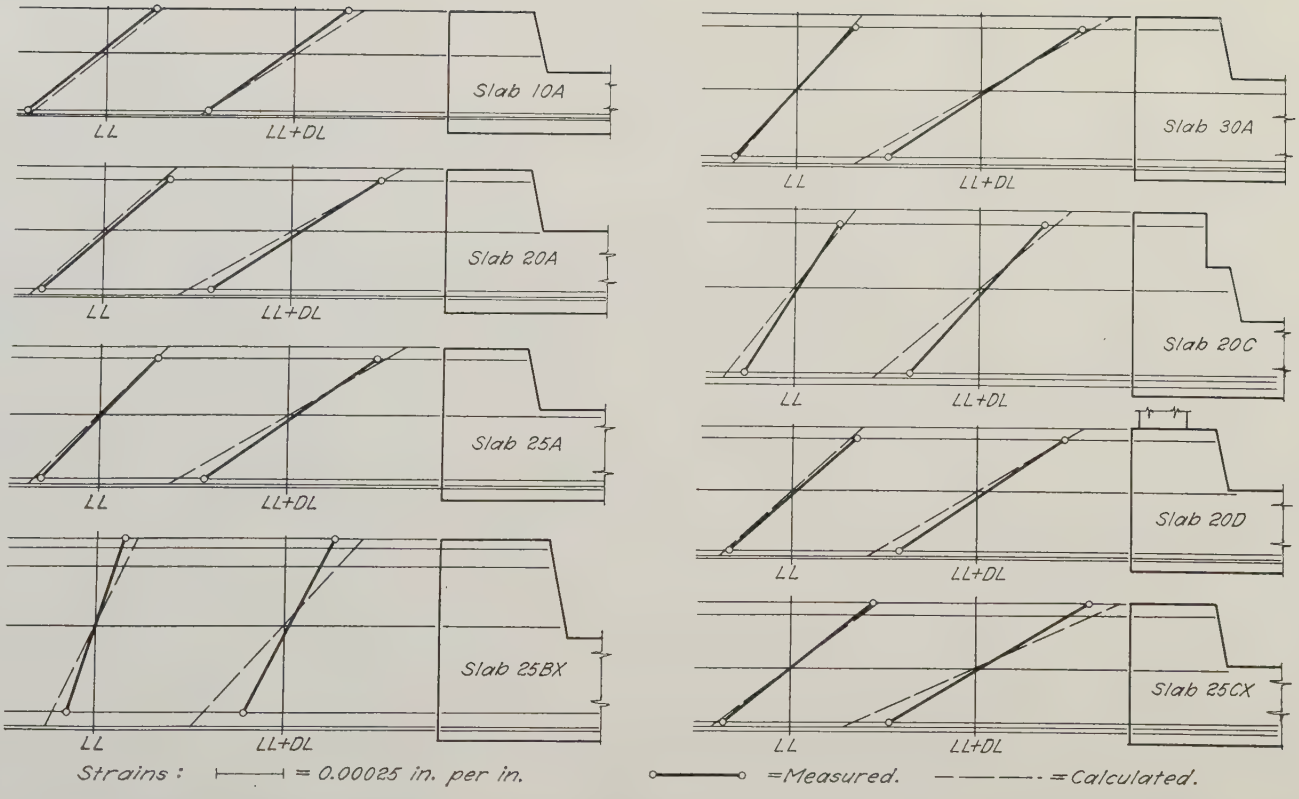


FIG. 36. MEASURED STRAINS IN CURB COMPARED WITH STRAINS CALCULATED BY SIMPLIFIED METHOD

Table 6 shows dimensional data pertaining to the slabs, and gives numerical values of the constants C_1, C_2, and C_3 as well as of the live and dead loads. Table 7 gives the calculated moments due to live and dead loads separately, and due to live plus dead load. Figure 36 shows the distribution of strains in the curbs as calculated for the composite curb sections, and as determined by measurement. Results for live load only are plotted separately from those for live plus dead load. It may be observed that the agreement in the strains for live load is good in most instances. The greatest divergence appears in specimens 20C and 25BX in which the curbs were stiffest. Strains obtained by test for applied distributed loads, and converted to full dead load effects, are generally smaller than the calculated values. Nevertheless, the combined effects of dead and live load are believed to be sufficiently well represented by the simplified analysis to warrant the use of the simplified method for design.

Strains in the longitudinal steel at the center of the slabs, calculated from moments determined for live load according to Equation (9), are compared with measured values in Fig. 17. The agreement is better for each slab than it was for strains calculated according to the analysis of an elastic slab.

The resisting moment of each slab at initial yielding of the longitudinal steel was calculated by the ordinary formula

$$M_r = A_s f_s jd$$

for a stress

$$f_s = \text{yield point} = 48\,000 \text{ psi.}$$

The moment in the slab due to dead load was subtracted from the resisting moment in each case, and the result was divided by the moment for a single live load. The quotient, given in column 8 of Table 7, represents the number of live loads calculated to produce initial failure of the slab. The capacity of the slabs calculated in this way is fairly reliable as indicated by a comparison with the test results given in column 9 of the table.

20. *Re-design of Structures for 10-, 20-, and 30-ft. Span by Simplified Method.*—Structures having the curb detail and spans previously specified for slabs 10A, 20A, and 30A were re-designed by the simplified method. In line with the discussion given in the first two paragraphs of Section 17, the slabs were re-designed to support 1.2 DL + 1.5 LL at a yield point of 40 000 psi. in the reinforcement. In select-

TABLE 8
DATA FOR RE-DESIGNED STRUCTURES

Factor		Slab 10A-R	Slab 20A-R	Slab 30A-R	Units
Span		10	20	30	ft.
Total depth of slab h		8.5	11.5	15	in.
Wheel load P		21 930	21 520	21 160	lb.
Unit load p		106	144	188	lb./ft.^{2}
Curb load q		298	348	407	lb./ft.
Stiffness factor $G = \frac{1}{H}$		0.776	2.45	5.18	....
Coefficients	C_1 Eq. (3)	0.599	0.723	0.618	....
	C_2 Eq. (4)	0.1453	0.1772	0.1605	....
	C_3 Eq. (5)	0.746	0.510	0.375	....
$(M_{slab})_{LL}$ Eq. (9)		7 080	13 100	20 300	ft. lb./ft.
$(M_{slab})_{DL}$ Eq. (10)		1 020	5 360	16 750	
$0.6(M_{slab})_{DL} + 1.25(M_{slab})_{LL}$		9 460	19 590	35 400	
$(M_{L\text{-}curb})_{LL}$ Eq. (11)		52 900	127 900	199 700	ft. lb.
$(M_{L\text{-}curb})_{DL}$ Eq. (12)		10 290	60 000	182 400	
$0.6(M_{L\text{-}curb})_{DL} + 1.25(M_{L\text{-}curb})_{LL}$		72 300	195 900	359 100	
Longitudinal reinforcement in bottom of slab	Bars	¾-in. rd.– 5.5 in. o.c.	1-in. rd.– 7 in. o.c.	1-in. rd.– 5.25 in. o.c.	....
	A_s	0.96	1.35	1.81	in.^{2}/ft.
	p	0.0114	0.0113	0.0112	....
Reinforcement in top of curb	Bars	None	4–1-in. sq.	2–1⅛-in. sq. 6–1-in. sq.	....
	A_s'	0	4.00	8.54	in.^{2}
	p'	0	0.0147	0.0265	....
Reinforcement in bottom of curb	Bars	4–¾-in. rd.	2–1-in. sq. 2–1-in. rd.	2–1⅛-in. sq. 2–1-in. sq.	....
	A_s	1.76	3.58	4.54	in.^{2}
	p	0.0076	0.0131	0.0141	....
Resisting moment of slab at $Y.P. = 40\,000$		19 600	39 400	71 900	ft. lb./ft.
Capacity of slab in number of live loads (in addition to DL)		2.62	2.60	2.72	....

ing the factor 1.2 for DL, it was felt that a value of 1.0 was hardly justified, but that the factor should be small since dead loads are accurately known for this type of structure, with little chance for overload, and since measured dead load strains were consistently less than calculated values. The factor 2.5 for LL was chosen in the belief that it provides an adequate factor of safety for live load.

On the basis of proportionality between load and stress, design according to the preceding requirements is equivalent to designing for 0.6 DL + 1.25 LL at a working stress of 20 000 psi. in the reinforcement. Such a method of design is not new, and has been discussed recently by A. J. Boase.* Designs were carried out in this manner using a working stress of 1200 psi. in the concrete and $n = 10$. The impact factor has been taken in accordance with the 1935 A.A.S.H.O. Standard Specifications for Highway Bridges in order that comparisons with the previous designs will not reflect a change in live load.

*"Saving Reinforcing Steel by Rationalizing Safety Factors," A. J. Boase, Engineering News-Record, May 7, 1942.

Table 8 gives results for the re-designed structures which are designated as 10A-R, 20A-R, and 30A-R for the spans of 10, 20, and 30 ft., respectively. A comparison with the previous designs shows that the depth of slab has been reduced 1.5 in. for the 10-ft. span, and 2 in. for the 20- and 30-ft. spans.

It is of interest to note in Table 8 that the moment in the slab for 0.6 DL + 1.25 LL is greater in slab 10A-R, about the same in slab 20A-R, and less in slab 30A-R than for DL + LL. Moments in the composite curb sections for live load, dead load, and design load are given in the table, as well as the principal requirements for the reinforcement of the slabs and curbs.

In determining the reinforcement required for the curb of slab 10A-R the tensile stress was decreased to 15 000 psi. At this stress no compression steel was required, and the percentage of tensile steel in the curb itself remained less than one per cent. In the composite curb the percentage of tensile steel is, of course, much higher, since it includes a portion of the slab reinforcement without any change in the width of the section in compression.

Reinforcement was determined for the curbs of slabs 20A-R and 30A-R on the basis of 16 000 psi. in the tension steel and using the specifications of the Joint Committee when calculating the required area of compressive steel. The final designs for these curbs are quite similar to those previously described in Table 2 for the prototypes of slabs 20A and 30A.

Finally, in Table 8 the capacity of each slab is given in terms of live loads required in addition to full dead load to produce a stress of 40 000 psi. in the reinforcement of the slab. If the actual yield point of the reinforcement is 45 000 psi. these capacities become 2.97, 2.97, and 3.16 live loads instead of 2.62, 2.60, and 2.72 for the slabs of 10-, 20-, and 30-ft. span, respectively.

21. *Design of Structure Having Unlike Curbs on Opposite Sides.*— The analysis given in Bulletin 315, and upon which Equations (3), (4), and (5) are based, is an analysis of an elastic slab having identical stiffening beams on two opposite edges. In Section 18 the moments obtained from the elastic analysis were combined with the total moments obtained by statics in such a way as to give reasonable agreement with tests of slab-bridges having integral, but identical, curbs. It is now desired to consider the modifications which must be made in the design method when the curbs are unlike.

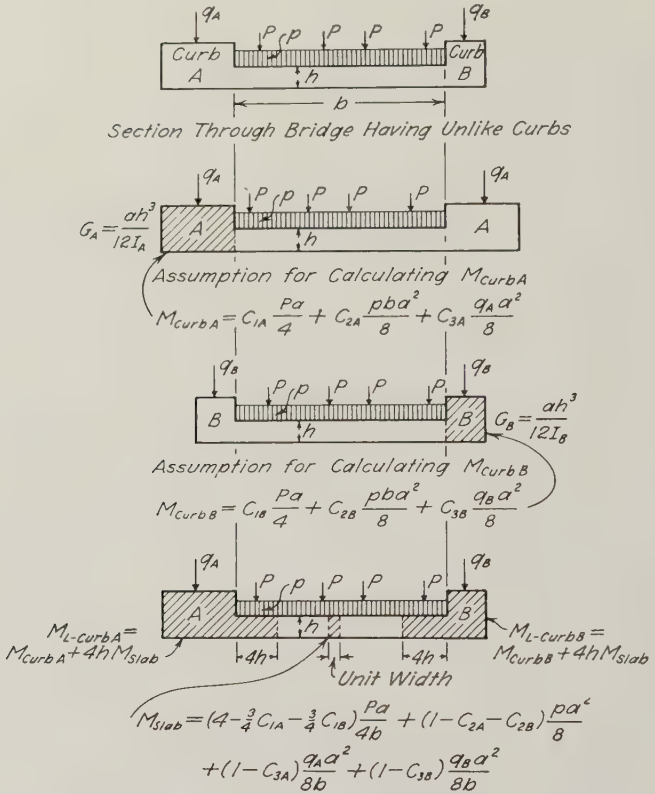


FIG. 37. APPROXIMATIONS FOR DESIGN OF BRIDGE HAVING UNLIKE CURBS

Figure 37 illustrates the essential steps which are made in determining the equations for the design of the slab and of the composite curbs. In making these steps it is recognized that the moment in a particular curb depends principally on the relative stiffness of that curb, the load on that curb, and the load on the slab. The relative stiffness of the other curb and the load on the other curb have little effect. Consequently, when calculating the moment in curb A the bridge is considered to have both curbs built and loaded as for curb A. This gives

$$M_{\text{curb A}} = C_{1A}\frac{Pa}{4} + C_{2A}\frac{pba^2}{8} + C_{3A}\frac{q_A a^2}{8} \qquad (13)$$

where C_{1A}, C_{2A}, and C_{3A} are the coefficients obtained from Equations (3), (4), and (5) when using a coefficient of stiffness for curb A, namely,

$$G_A = \frac{ah^3}{12\,I_A}.$$

For curb B the bending moment is similarly

$$M_{\text{curb B}} = C_{1B}\frac{Pa}{4} + C_{2B}\frac{pba^2}{8} + C_{3B}\frac{q_B a^2}{8}. \tag{14}$$

The total bending moment, from statics, is

$$M_{\text{statics}} = m\frac{Pa}{4} + \frac{pba^2}{8} + \frac{q_A a^2}{8} + \frac{q_B a^2}{8} \tag{15}$$

where m is the number of wheel loads on the bridge. By allowing, as before, a 25 per cent reduction in the live load curb moments due to a shifting of the wheel loads, the moment to be resisted by the entire slab may be found as

$$M = \left(m - \frac{3}{4}C_{1A} - \frac{3}{4}C_{1B}\right)\frac{Pa}{4} + (1 - C_{2A} - C_{2B})\frac{pba^2}{8} + (1 - C_{3A})\frac{q_A a^2}{8} + (1 - C_{3B})\frac{q_B a^2}{8}. \tag{16}$$

The average moment per unit width of slab is then

$$M_{\text{slab}} = \left(m - \frac{3}{4}C_{1A} - \frac{3}{4}C_{1B}\right)\frac{Pa}{4b} + (1 - C_{2A} - C_{2B})\frac{pa^2}{8} + (1 - C_{3A})\frac{q_A a^2}{8b} + (1 - C_{3B})\frac{q_B a^2}{8b}. \tag{17}$$

This may readily be divided into two parts to represent separately the moments due to live load and to dead load. Thus one obtains

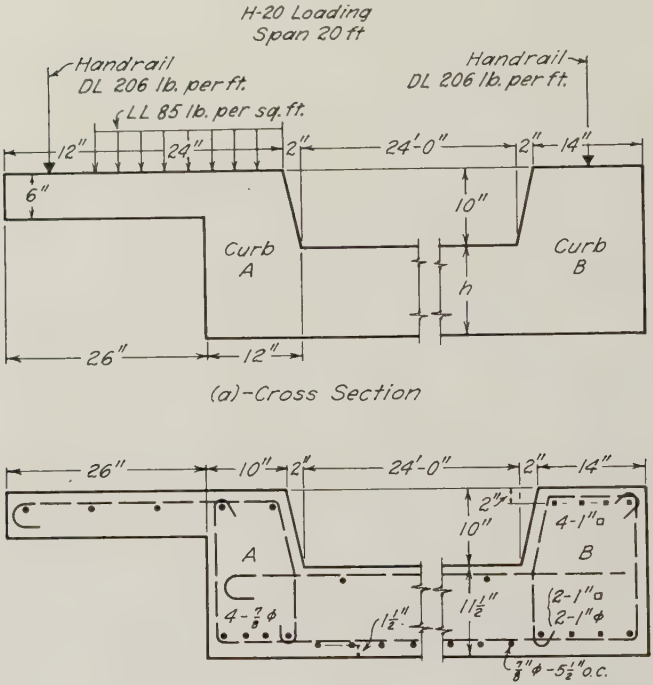


FIG. 38. SECTION THROUGH BRIDGE HAVING UNLIKE CURBS

$$(M_{\text{slab}})_{\text{LL}} = \left(m - \frac{3}{4}C_{1\text{A}} - \frac{3}{4}C_{1\text{B}}\right)\frac{Pa}{4b} + (1 - C_{3\text{A}})\frac{(q_{\text{A}})_{\text{LL}}\,a^2}{8b} + (1 - C_{3\text{B}})\frac{(q_{\text{B}})_{\text{LL}}\,a^2}{8b} \tag{18}$$

$$(M_{\text{slab}})_{\text{DL}} = (1 - C_{2\text{A}} - C_{2\text{B}})\frac{pa^2}{8} + (1 - C_{3\text{A}})\frac{(q_{\text{A}})_{\text{DL}}\,a^2}{8b} + (1 - C_{3\text{B}})\frac{(q_{\text{B}})_{\text{DL}}\,a^2}{8b}. \tag{19}$$

These equations have been written to cover the general case in which a live load may be supported on a sidewalk curb. As in the procedure described in Section 18, the design of the slab is made by trial.

Following the design of the slab on the basis of moments calculated

from Equations (18) and (19), the reinforcement in the composite sections of curb and slab must be determined. For this purpose a portion of the slab of width $4h$ is again assumed to act with the curb. The moments in the composite section for curb A are calculated from the equations

$$(M_{\text{L-curb A}})_{\text{LL}} = C_{1\text{A}} \frac{Pa}{4} + C_{3\text{A}} \frac{(q_{\text{A}})_{\text{LL}}\, a^2}{8} + 4h\,(M_{\text{slab}})_{\text{LL}} \qquad (20)$$

$$(M_{\text{L-curb A}})_{\text{DL}} = C_{2\text{A}} \frac{pba^2}{8} + C_{3\text{A}} \frac{(q_{\text{A}})_{\text{DL}}\, a^2}{8} + 4h\,(M_{\text{slab}})_{\text{DL}}. \qquad (21)$$

Similar equations, with a subscript B in place of A, apply to curb B. Reinforcement in the curbs may now be determined.

As an application of the method to the design of a slab-bridge having unlike curbs, a bridge having the cross-section shown in Fig. 38(a) was designed for H-20 loading on a span of 20 ft. In this bridge curb A is a sidewalk curb and curb B is identical with the ordinary curbs of the test structures. For the sidewalk curb it was assumed that the cantilever sidewalk and the upper part of the curb would be adequately doweled to the slab so as to act as a unit. It is worth-while to make this provision in order to obtain the load-carrying capacity of the curb as a structural member.

The design was carried out in the manner described in Section 20 for 0.6 DL + 1.25 LL at working stresses of 20 000 psi. in the reinforcement and 1200 psi. in the concrete using $n = 10$. The impact factor was taken in accordance with the 1941 A.A.S.H.O. Standard Specifications for Highway Bridges. A summary of the principal results of the design is given in the following:

Design of Bridge With Unlike Curbs (see Fig. 38)

0.6 DL + 1.25 LL at f_s = 20 000 psi.; f_c = 1200 psi.; n = 10.
Span, a = 20 ft. = 240 in.
Width of roadway, b = 24 ft.
H-20 Loading (Impact factor = 0.300)
P = 16 000 (1.300) = 20 800 lb. per wheel
m = 4 wheels on bridge
Total depth of slab, h = 11.5 in. (d = 10.0 in.)
p = 144 lb./ft.^{2}
Handrails, 206 lb./ft.

Curb A:

$$G_A = \frac{ah^3}{12\,I_A} = \frac{240\,(11.5)^3}{12\,(15\,240)} = 2.00$$

$C_{1A} = 0.796$; $C_{2A} = 0.1947$; $C_{3A} = 0.561$

$(q_A)_{LL} = 170$ lb./ft.; $(q_A)_{DL} = 626$ lb./ft. (includes handrail).

Curb B:

$$G_B = \frac{ah^3}{12\,I_B} = \frac{240}{15}\left(\frac{11.5}{21.5}\right)^3 = 2.45$$

$C_{1B} = 0.723$; $C_{2B} = 0.1772$; $C_{3B} = 0.510$

$(q_B)_{LL} = 0$; $(q_B)_{DL} = 554$ lb./ft. (includes handrail).

Slab:

$(M_{slab})_{LL} = 12\,540$ ft. lb./ft. Equation (18)

$(M_{slab})_{DL} = 5\,660$ ft. lb./ft. Equation (19)

$M_{slab} = 0.6\,(M_{slab})_{DL} + 1.25\,(M_{slab})_{LL} = 19\,070$ ft. lb./ft.

$A_s = 1.31$ in.^{2}/ft. of slab

$\frac{7}{8}$-in. round bars, $5\frac{1}{2}$-in. o.c.

Composite Curb A:

$(M_{L\text{-curb A}})_{LL} = 135\,600$ ft. lb. Equation (20)

$(M_{L\text{-curb A}})_{DL} = 72\,900$ ft. lb. Equation (21)

$M_{L\text{-curb A}} = 0.6\,(M_{L\text{-curb}})_{DL} + 1.25\,(M_{L\text{-curb}})_{LL} = 213\,200$ ft. lb.

No compression reinforcement needed

Tension reinforcement, $A_s = 2.40$ in.^{2}

$4 - \frac{7}{8}$-in. round bars

(calculated for $f_s = 20\,000$ psi.)

Composite Curb B:

$(M_{L\text{-curb B}})_{LL} = 123\,300$ ft. lb. Equation (20), B replaces A

$(M_{L\text{-curb B}})_{DL} = 66\,400$ ft. lb. Equation (21), B replaces A

$M_{L\text{-curb B}} = 0.6\,(M_{L\text{-curb B}})_{DL} + 1.25\,(M_{L\text{-curb B}})_{LL} = 194\,000$ ft. lb.

Compression reinforcement, 4 — 1-in. square bars

$A_s = 4.00$ in.^{2}

Tension reinforcement, 2 — 1-in. square bars
2 — 1-in. round bars
$A_s = 3.58$ in.^{2}
(calculated for $f_s = f_s' = 16\,000$ psi.)

Details of the principal reinforcement are shown in Fig. 38(b). The similarity of curb B and the curbs of slab 20A-R (Table 8) may be noted.

22. *Structures Having Spans Greater Than 30 Feet.*—For spans greater than 30 ft. the effect of the front wheel loads becomes of sufficient importance to be accounted for in design. The added moment due to front wheel loads cannot be expected to be apportioned among the curbs and slab in exactly the same way as the moment due to the rear wheels. However, this additional moment is a small part of the total design moment, and a considerable error in its distribution is of little consequence as long as provision is made for its total magnitude.

A simple method of providing for the added total moment due to the front wheels is to use the equations given for the simplified method of design except that P be increased to KP where K is a multiplication factor which is defined as the ratio of the maximum total live load moment in the bridge when loaded with front and rear wheels to the total moment in the bridge when loaded only by the rear wheels at mid-span. With the front wheels carrying one-quarter as much load as the rear wheels, and with the distance between axles defined as s, the ratio may be shown to be expressed exactly by the formula

$$K = \frac{1}{20}\left(5 - \frac{s}{a}\right)^2. \tag{22}$$

For example, with $s = 14$ ft. from the A.A.S.H.O. specifications and with a span of 40 ft.,

$$K = \frac{1}{20}\left(5 - \frac{14}{40}\right)^2 = 1.08.$$

The coefficients C_1, C_2, and C_3 given by Equations (3), (4), and (5) are believed to be reliable for spans up to 45 ft. Furthermore, in the simplified method of design, the formulas for calculating bending

moments are based upon the total bending moment which is determined by statics. The only extrapolation involved in applying the formulas to spans greater than 30 ft. is therefore concerned with the division of moment between the slab and the curbs. In this respect there is a considerable range in which the strength of the structure as a whole will not be affected.

From Table 8 it will be noted that the compressive reinforcement in the curb becomes increasingly difficult to provide as the span increases, and the cross-section of the curb remains constant above the roadway surface. This is to be expected, since a small section of concrete projecting above a very heavy slab tends to undergo excessive deformations when the slab bends. A small area projecting above the roadway surface becomes incapable of materially raising the neutral surface of a very heavy slab. The structural importance of the curb has been demonstrated in the tests, and is confirmed by analysis. It is reasonable, then, to increase the cross-sectional area of the curb for the bridges of longer span in order that the curb may continue to function as a load-carrying member. On a 40-ft. span a curb projecting 10 in. above the roadway might reasonably have a width of 18 or 20 in. at the top instead of 14 in. as for the bridges discussed in Section 20. A stepped section might also be used so as to provide added depth.

Figure 39 shows a possible detail of curb and slab for a bridge of 40-ft. span and 26-ft. roadway designed for the following conditions:

H-20 loading (from Equation (22), $K = 1.08$)
Allowance for handrail, 206 lb./ft.
Impact factor = 0.300
1.2 DL + 2.5 LL at $YP = 40\,000$ psi. in steel
Design, 0.6 DL + 1.25 LL at $f_s = 20\,000$; $f_c = 1200$; $n = 10$.

23. *Truck-Trailer Loadings.*—The 1941 Standard Specifications for Highway Bridges of the A.A.S.H.O. define certain H-S loadings to cover the case of a tractor truck with semi-trailer. For such loadings it is recommended that the simplified method of design be used except that P be increased to TP where T is a multiplication factor which is defined as the ratio of the maximum total live load moment in the bridge when loaded with the truck-trailer loading to the total live load moment in the bridge when loaded only by a single line of wheel loads at mid-span. For spans between 24 ft. and 34 ft. the loading is equal on two axles spaced at a distance s of 14 ft. For spans greater than

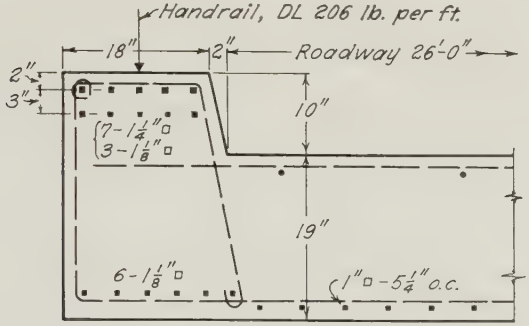


FIG. 39. CURB DETAIL FOR BRIDGE OF 40-FT. SPAN

34 ft. the loading must also include the front wheel loads which are one-quarter as great as the rear wheel loads, but are separated from them by the same axle spacing $s = 14$ ft. It may be shown that, for spans between 24 ft. and 34 ft.,

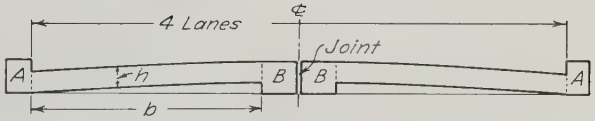
$$T = \frac{1}{2}\left(2 - \frac{s}{a}\right)^2 \tag{23-a}$$

and, for spans between 34 ft. and 45 ft.,

$$T = \frac{1}{4}\left(9 - \frac{s}{a}\right)\left(1 - \frac{s}{a}\right). \tag{23-b}$$

24. *Wide Bridge Having Center Longitudinal Joint.*—A wide bridge is sometimes constructed with a center longitudinal joint, but without a center curb projecting above the slab to separate traffic. Such a bridge may have a cross-section of the general type represented in Fig. 40. In this cross-section a portion of the slab at the joint has been thickened, producing, in effect, a shallow inverted curb. When this is done the slab may be designed by the use of Equations (18) and (19) in exactly the same manner as described in Section 21 for structures having unlike curbs. The width of slab, b, is taken as the distance between the inside faces of the curbs, as shown in Fig. 40.

The reinforcement in curb A, the normal curb, may be determined as described in Section 21. For curb B, the inverted curb, a special problem arises because (1) the wheel loads may be shifted toward the center of the bridge until one wheel is directly over the curb, and (2) the exact manner in which interaction occurs between the slab and

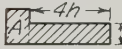


$G_A = \frac{ah^3}{12I_A}$ $G_B = \frac{ah^3}{12I_B}$ C_1 Eq. (3) C_3 Eq. (5) C_2 Eq. (4) C_4 Eq. (25)

Slab: $\frac{1}{2}h$ Unit Width

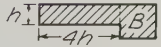
$$(M_{Slab})_{LL} = (4 - \tfrac{3}{4}C_{1A} - \tfrac{3}{4}C_{1B})\frac{Pa}{4b} + (1 - C_{3A})\frac{(q_A)_{LL}a^2}{8b}$$

$$(M_{Slab})_{DL} = (1 - C_{2A} - C_{2B})\frac{pa^2}{8} + (1 - C_{3A})\frac{(q_A)_{DL}a^2}{8b} + (1 - C_{3B})\frac{q_B a^2}{8b}$$

Composite Curb A: 

$$(M_{L\text{-}curb\,A})_{LL} = C_{1A}\frac{Pa}{4} + C_{3A}\frac{(q_A)_{LL}a^2}{8} + 4h(M_{Slab})_{LL}$$

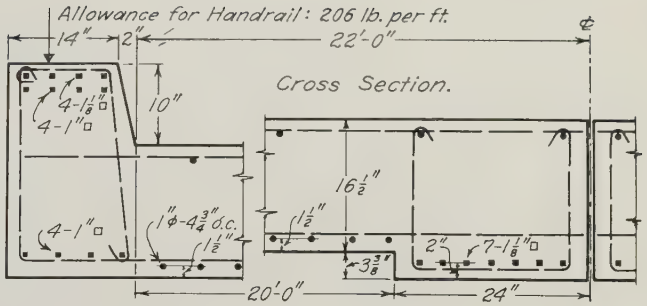
$$(M_{L\text{-}curb\,A})_{DL} = C_{2A}\frac{pba^2}{8} + C_{3A}\frac{(q_A)_{DL}a^2}{8} + 4h(M_{Slab})_{DL}$$

Composite Curb B: 

$$(M_{L\text{-}curb\,B})_{LL} = C_{4B}\frac{Pa}{4} + 4h(M_{Slab})_{LL}$$

$$(M_{L\text{-}curb\,B})_{DL} = C_{2B}\frac{pba^2}{8} + C_{3B}\frac{q_B a^2}{8} + 4h(M_{Slab})_{DL}$$

FIG. 40. CROSS-SECTION AND SUMMARY OF FORMULAS; WIDE BRIDGE HAVING CENTER LONGITUDINAL JOINT AND INVERTED CURB



Design Data:
Span $a = 30$ ft. H-20 Loading;
$P = 20\,800$ lb., Including Impact; $p = 206$ lb. per sq. ft.
$0.6DL + 1.25LL$ at $f_s = 20\,000$ psi., $f_c = 1200$ psi., $n = 10$.

Curb A:		Curb B:	
$q_A = 620$ lb. per ft.	$C_{2A} = 0.1714$	$q_B = 498$ lb. per ft.	$C_{2B} = 0.1322$
$G_A = 5.79$	$C_{3A} = 0.378$	$G_B = 8.55$	$C_{3B} = 0.292$
$C_{1A} = 0.573$		$C_{1B} = 0.430$	$C_{4B} = 0.561$

Slab:
$(M_{Slab})_{DL} = 20\,300$ ft. lb. per ft. $(M_{Slab})_{LL} = 25\,300$ ft. lb. per ft.
$0.6(M_{Slab})_{DL} + 1.25(M_{Slab})_{LL} = 43\,800$ ft. lb. per ft.

Composite Curb A:
$(M_{L\text{-}curb\,A})_{DL} = 217\,400$ ft. lb. per ft. $(M_{L\text{-}curb\,A})_{LL} = 228\,700$ ft. lb. per ft.
$0.6(M_{L\text{-}curb\,A})_{DL} + 1.25(M_{L\text{-}curb\,A})_{LL} = 416\,400$ ft. lb. per ft.

Composite Curb B:
$(M_{L\text{-}curb\,B})_{DL} = 189\,200$ ft. lb. per ft. $(M_{L\text{-}curb\,B})_{LL} = 226\,800$ ft. lb. per ft.
$0.6(M_{L\text{-}curb\,B})_{DL} + 1.25(M_{L\text{-}curb\,B})_{LL} = 397\,000$ ft. lb. per ft.

FIG. 41. DETAILS FOR WIDE BRIDGE HAVING CENTER LONGITUDINAL JOINT AND INVERTED CURB

an inverted curb is not known from tests. To solve the first problem of the increased live load moment in the curb due to the shifting of the loads, an equation may be used in the form

$$(M_{\text{curb}})_{\text{LL}} = C_4 \frac{Pa}{4} \tag{24}$$

where C_4 is given, for spans varying from 5 to 45 ft., by the empirical equation

$$C_4 = \frac{1 + 0.015\,G}{1 + 0.35\,G} + \left(\frac{13}{2.5 + G}\right)\frac{3 - 2\,(v/a)}{3 + 40\,(v/a)}. \tag{25}$$

In Equation (25) the first term accounts for the single load directly over the curb and the remaining part accounts for the other three loads on the slab. In this equation the factor G and the dimensions v and a are as defined in Section 18. This equation has been determined on the basis of the analysis given in Bulletin 315.

It should be noted that the coefficient C_4 enters the analysis only when the bending moment in the inverted curb is to be determined. For the design of the slab the loads are shifted back over the slab and the coefficients and equations for moment are as given in previous sections.

Concerning the second problem, that of interaction between the curb and slab, in the absence of test data it seems reasonable to adopt the same procedure for the inverted curb as was used for the normal curb. That is, a composite section will be considered in which a portion of the slab of width $4h$ will be assumed to act with the curb. In this composite section, therefore, the live load moment is

$$(M_{\text{L-curb}})_{\text{LL}} = C_4 \frac{Pa}{4} + 4h\,(M_{\text{slab}})_{\text{LL}}\,. \tag{26}$$

The moment due to dead load is, as before, given by Equation (12).

To summarize, the wide bridge having a longitudinal joint and an inverted curb at the joint may be designed in the same manner as described in Section 21 for the bridge having unlike curbs, with the sole difference that C_4 replaces C_1 in the equation for the live load moment in the inverted curb. A summary of the equations is given below the cross-section of the bridge in Fig. 40.

Figure 41 gives details for a wide bridge of 30-ft. span designed for H-20 loading on two lanes on each side of a central longitudinal joint. A shallow inverted curb is used at the joint to strengthen the edge of the slab at the joint. The design was carried out for the same loads and stresses as for the bridge with unlike curbs described in Section 21. It may be observed that no compression reinforcement is required in the inverted curb but that a large amount of tensile reinforcement is required. The reason for the large requirement of tensile reinforcement is fairly obvious; in the composite section at the curb the part of the reinforcement in the slab is forced to act at a lower efficiency when the governing stress is that which occurs in the bottom reinforcement of the curb. That is, a projection below the slab tends to be strained excessively, and it requires a large amount of reinforcement to overcome this tendency.

25. *Summary of Simplified Method of Design.*—For slab-bridges having two or more lanes of traffic a simplified method of design is described as follows:

(a) *Bridges With Wheel Loads at Mid-Span Only.*—With the notation defined in Sections 18 and 21, and using a trial depth of slab, one may compute live and dead load moments per unit width of slab from the equations

$$(M_{\text{slab}})_{\text{LL}} = \left(m - \frac{3}{4}C_{1\text{A}} - \frac{3}{4}C_{1\text{B}}\right)\frac{Pa}{4b} + (1 - C_{3\text{A}})\frac{(q_{\text{A}})_{\text{LL}}\,a^2}{8b} + (1 - C_{3\text{B}})\frac{(q_{\text{B}})_{\text{LL}}\,a^2}{8b} \tag{18}$$

$$(M_{\text{slab}})_{\text{DL}} = (1 - C_{2\text{A}} - C_{2\text{B}})\frac{pa^2}{8} + (1 - C_{3\text{A}})\frac{(q_{\text{A}})_{\text{DL}}\,a^2}{8b} + (1 - C_{3\text{B}})\frac{(q_{\text{B}})_{\text{DL}}\,a^2}{8b}. \tag{19}$$

In these equations the coefficients are

$$C_{1\text{A, B}} = \left(\frac{12}{2.5 + G_{\text{A, B}}}\right)\frac{4 - (v/a)}{4 + 28\,(v/a)} \tag{3'}$$

$$C_{2A,\,B} = \frac{0.5\,(a/b)}{0.47\,G_{A,\,B} + \sqrt[3]{1.15 + (a/b)^3}} \qquad (4')$$

$$C_{3A,\,B} = \frac{\sqrt[3]{1.15 + (a/b)^3}}{0.47\,G_{A,\,B} + \sqrt[3]{1.15 + (a/b)^3}} \qquad (5')$$

wherein the subscripts A and B refer to curb A and curb B, respectively, and

$$G_A = \frac{ah^3}{12\,I_A}, \qquad G_B = \frac{ah^3}{12\,I_B}.$$

The ratios v/a and a/b are defined in Section 18.

Final design of the slab permits the calculation of moments in composite sections at each curb. A composite section includes the curb and the adjacent width of slab equal to 4 times the total slab thickness. The live and dead load moments in the composite curb A are given by the equations

$$(M_{\text{L-curb A}})_{LL} = C_{1A}\frac{Pa}{4} + C_{3A}\frac{(q_A)_{LL}\,a^2}{8} + 4h\,(M_{\text{slab}})_{LL} \qquad (20)$$

$$(M_{\text{L-curb A}})_{DL} = C_{2A}\frac{pba^2}{8} + C_{3A}\frac{(q_A)_{DL}\,a^2}{8} + 4h\,(M_{\text{slab}})_{DL}. \qquad (21)$$

A similar pair of equations, with subscript B in place of A, applies to curb B. From these equations the moments to be resisted by the composite curbs may be computed and the reinforcement determined at once.

Transverse reinforcement may be provided at a nominal percentage such as 0.35 per cent of bd.

(b) *H-Loadings; Spans Greater Than 30 Feet.*—For spans between 30 and 45 ft. the effect of the front wheels on the bridge may be accounted for by replacing P by KP in the preceding equations, where

$$K = \frac{1}{20}\left(5 - \frac{s}{a}\right)^2. \qquad (22)$$

In this equation s/a is the ratio of the spacing between axles to the span of the slab.

(c) *H-S Loadings; Spans Greater Than 24 Feet.*—When the maximum total live load moment on the bridge is increased due to truck-trailer loadings, defined as H-S loadings in the 1941 A.A.S.H.O. Standard Specification for Highway Bridges, the effect of this increase may be provided for by replacing P by TP in the preceding equations, where T is the ratio of the maximum total live load moment to the moment calculated for a single line of loads on the centerline of the bridge. Expressed algebraically

$$T = \frac{1}{2}\left(2 - \frac{s}{a}\right)^2 \qquad \text{for spans between 24 ft. and 34 ft.} \quad (23\text{-a})$$

$$T = \frac{1}{4}\left(9 - \frac{s}{a}\right)\left(1 - \frac{s}{a}\right) \text{ for spans between 34 ft. and 45 ft.} \quad (23\text{-b})$$

(d) *Wide Bridges Having a Center Longitudinal Joint.*—For the 4-lane slab-bridge which is to be built with a center longitudinal joint but without a center curb above the roadway surface to separate traffic, a special problem arises in connection with the unsupported edge of the slab at the joint. This problem, discussed in detail in Section 24, may be solved in the following way:

The slab may be thickened at the joint by adding depth below the bottom surface of the slab. This provides, in effect, an inverted curb at the joint. This curb is subject to unusually high live load moment since truck wheels may pass directly over it. A special coefficient,

$$C_4 = \frac{1 + 0.015\,G}{1 + 0.35\,G} + \left(\frac{13}{2.5 + G}\right)\frac{3 - 2\,(v/a)}{3 + 40\,(v/a)},$$

is then to be used in the equation for live load moment in the curb,

$$(M_{\text{curb}})_{\text{LL}} = C_4\frac{Pa}{4}.$$

The slab and the normal curb may, however, be designed as before. A summary of the equations for design is given in Fig. 40, and a particular example is illustrated in Fig. 41.

(e) *Factor of Safety.*—A fairly constant factor of safety in terms of live load at initial failure of the bridge is provided by designing for 1.2 DL + 2.5 LL at the minimum specified yield point of the steel. If the yield point is 40 000 psi. this is equivalent to designing for 0.6 DL + 1.25 LL at a working stress of 20 000 psi.

It should be noted that the choice of working stresses or factor of safety does not in any way affect the simplified method of calculating bending moments. Designs for DL + LL at working stresses may be carried out without alteration of any of the formulas for moments.

V. Conclusions

26. *Conclusions.*—In Bulletin 315 an analysis was made of the simple span slab-bridge with stiffened edges. The analysis pertains strictly to a slab of elastic homogeneous material, simply supported on two opposite edges and stiffened on the other edges by beams which were constrained to deflect with the edges of the slab, but which were otherwise unaffected by the slab. Moments in the slab and in the edge beams were determined for three types of loading so as to include all of the loads on a highway slab-bridge having a span not greater than 30 ft.

On the basis of moments determined by the method prescribed in Bulletin 315, nine slab-bridges were designed for spans varying from 10 to 30 ft. and for varying stiffness of curb. Two half-scale and 7 quarter-scale models of these bridges were tested. The test specimens and the tests are described in detail in Chapters II and III of this bulletin. The tests are summarized and discussed in Sections 16 and 17.

The most significant conclusions to be drawn from the tests are the following:

(1) Influence lines for strains at the center of the slab and at mid-span of the curb were verified quantitatively by some tests and only qualitatively by others.

(2) At the center of the slab, after it had been systematically cracked, the measured strains in the principal (longitudinal) reinforcement showed better agreement with the calculated strains as the span increased, but were consistently smaller than the calculated values.

(3) At the center of the slab, after cracking, the measured strains in the transverse reinforcement were materially less than the calculated values.

(4) In some instances the measured compressive strains at the mid-span of the curb were considerably larger than the calculated

strains, in spite of the large quantity of compressive reinforcement which had been provided.

(5) In each of the curbs the effect of the slab on the curb was evident in a material lowering of the neutral axis of the curb at mid-span.

(6) In all slabs initial failure occurred by yielding of the longitudinal reinforcement.

(7) The capacity of the bridges in terms of live loads at initial failure appears to be more closely related to the average moment than to the maximum moment in the slabs.

(8) Before final failure the slabs tended to pull away from the curbs.

(9) A reserve strength, beyond the capacity at initial failure as defined in Section 15, generally of about 25 or 30 per cent of the applied live load at initial failure, was exhibited by the structures.

On the basis of the test results a modified method of design was evolved. This method makes use of three empirical equations which are based on the analysis given in Bulletin 315. The method is described fully in Chapter IV and is summarized in Section 25. The scope of design is extended in the simplified method to include bridges with unlike curbs, bridges having a center longitudinal joint, spans up to 45 ft., and truck-trailer loadings. The principal features of the simplified method are that it provides a total resisting moment which is determined mainly by statics, and that it requires the reinforcement in the curb to be determined for a composite section which includes an adjoining portion of the slab.

Calculations based on the simplified method show improved agreement with the test results in the following respects: (a) capacity of the structure at initial failure, (b) strain in the principal reinforcement at the center of the slab, and (c) strains at the top and bottom of the curbs at mid-span.

Structures re-designed by the simplified method for the range of spans covered by the tests show savings in the required thickness of slabs.

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